Propagative broad learning for nonparametric modeling of ambient effects on structural health indicators

Sin-Chi Kuok¹,², Ka-Veng Yuen¹, Stephen Roberts³ and Mark A Girolami²,⁴

Abstract
In this article, a novel propagative broad learning approach is proposed for nonparametric modeling of the ambient effects on structural health indicators. Structural health indicators interpret the structural health condition of the underlying dynamical system. Long-term structural health monitoring on in-service civil engineering infrastructures has demonstrated that commonly used structural health indicators, such as modal frequencies, depend on the ambient conditions. Therefore, it is crucial to detrend the ambient effects on the structural health indicators for reliable judgment on the variation of structural integrity. However, two major challenging problems are encountered. First, it is not trivial to formulate an appropriate parametric expression for the complicated relationship between the operating conditions and the structural health indicators. Second, since continuous data stream is generated during long-term structural health monitoring, it is required to handle the growing data efficiently. The proposed propagative broad learning provides an effective tool to address these problems. In particular, it is a model-free data-driven machine learning approach for nonparametric modeling of the ambient-influenced structural health indicators. Moreover, the learning network can be updated and reconfigured incrementally to adapt newly available data as well as network architecture modifications. The proposed approach is applied to develop the ambient-influenced structural health indicator model based on the measurements of 3-year full-scale continuous monitoring on a reinforced concrete building.

Keywords
Structural health monitoring, propagative broad learning, nonparametric modeling, modal frequencies, ambient conditions

Introduction
Structural health monitoring of civil engineering infrastructures has attracted extensive attention for decades.¹⁻⁷ The goal of structural health monitoring is to investigate the health condition of a structure so that structural damage can be diagnosed for proper structural maintenance and safety assessment.⁸⁻¹³ A structural health indicator is a quantitative measure to enable the interpretation of the health condition of the monitored structure. However, long-term structural health monitoring of in-service infrastructures demonstrated that the structural health indicators are sensitive to varying operating conditions.⁸⁻¹⁴⁻¹⁶ Inevitable operating interferences, such as temperature and humidity,¹⁷⁻²¹ wind velocity,²²⁻²⁵ traffic loading,²⁶,²⁷ and base excitation,²⁸ can induce significant contributions to the variation of structural health indicators. In order to make reliable judgments of the deterioration of structural integrity, modeling the ambient interference-induced variation of the structural health indicators is essential.

However, due to the complex nature and uncertain mechanisms, formulating a representative parametric
model to describe these ambient effects on structural
health indicators is a challenging task for general civil
engineering infrastructures. Furthermore, an inap-
propriate parametric model can induce misleading and
biased interpretation.\textsuperscript{29–31} Bypassing the formulation of
a prescribed parametric model, nonparametric modell-
ing in machine learning establishes via learning the
input–output relationship based on attributions of the
data. In recent years, increasing research activities have
been devoted to the development of machine learning


techniques for structural health monitoring. Lam
et al.,\textsuperscript{32} Yuen and Lam,\textsuperscript{33} Zhou et al.,\textsuperscript{34} and Neves
et al.\textsuperscript{35} presented artificial neural network–based
approaches for structural integrity assessment. Rafiei
and Adeli\textsuperscript{36} proposed a neural dynamics classification
algorithm to detect and classify the severity of struc-
tural damage. Cha et al.\textsuperscript{37} and Bao et al.\textsuperscript{38} presented
deep learning–based approaches for vision-based struc-
tural damage detection. Zhao et al.\textsuperscript{39} employed deep
neural networks for strain-based damage identification.
Pan et al.\textsuperscript{40} applied deep Bayesian belief network
learning to evaluate the structural health state. Huang
et al.\textsuperscript{41} adopted sparse Bayesian learning for fractal
dimension analysis of beam and bridge structures.
Huang et al.\textsuperscript{42} incorporated multi-task sparse Bayesian
learning and hierarchical models for structural damage
detection and missing data recovery. Regarding the
ambient effects on the structural behavior, Figueiredo
et al.\textsuperscript{43} compared the performance of four machine
learning algorithms on analyzing the structural beha-
vior of a laboratory frame structure under changing
operational conditions. Mu and Yuen\textsuperscript{44} presented an
updated sparse Bayesian learning approach for model-
ing the modal frequency–environmental condition rela-
tionship. Kang et al.\textsuperscript{45} applied the kernel extreme
learning machine technique to model the relation
between the structural response of dams and opera-
tional conditions including temperature, reservoir water
level, and permanent deformation. Ding et al.\textsuperscript{46} pre-

dented a sparse deep belief network for structural dam-
age identification by utilizing limited noisy data.
Adeli,\textsuperscript{47} Worden and Manson,\textsuperscript{48} Farrar and Worden,\textsuperscript{49}
and Bao et al.\textsuperscript{5} provided comprehensive reviews on the
development, implementation, and future trend of
machine learning in structural health monitoring.

In this study, propagative broad learning (PBL) is
proposed as an approach to reveal the ambient effects
on the variation of the identified structural health indi-
cators. PBL provides an effective tool for nonpara-
metric data-driven modeling with growing data. The
proposed learning network extends the recently devel-
oped broad learning system\textsuperscript{50,51} to handle the growing
training data generated in long-term structural health
monitoring. The broad learning system offers a computu-
tationally efficient alternative to deep learning

architectures and is designed under the functional link
neural network with a rank-expansion learning
scheme.\textsuperscript{52,53} By leveraging the expandable characteris-
tics of the broad learning system, the learning network
of the proposed PBL is updated in a propagative man-
er. For every time step, the trained network of the pre-
vious time step is utilized as the prior network. The
identified structural health indicators and the associated
ambient conditions of the current time step form the
new set of training data to update the learning network.
To enhance the data fitting capacity, the learning net-
work can be expanded to include additional compo-
nents for polishing the input–output relationship.
Previous studies showed that broad learning networks
achieve the same data fitting performance as deep learn-
ing networks.\textsuperscript{50,54} However, conventional deep learning
suffers from a time-consuming training process with
numerous parameters in the hierarchical layers.\textsuperscript{55,56} The
computational cost for developing conventional deep
learning neural networks to handle growing data in a
propagative manner can be very expensive or even pro-
hibitive. In contrast, the broad learning network can be
reconfigured incrementally to adapt the additional
training data and architecture modification. It allows
the PBL to efficiently handle the growing data gener-
ated in long-term structural health monitoring. To
demonstrate the efficacy of the proposed approach,
measurements from a 3-year, full-scale continuous
monitoring on a residential reinforced concrete building
are analyzed. In particular, the proposed approach is
applied to model the relationship between the identified
modal frequencies and the monitored ambient condi-
tions including temperature, relative humidity, and
wind velocity. In the following, the formulation of the
proposed PBL for nonparametric modeling of the
ambient interference on structural health indicators is
presented.

**Formulation of PBL to model the ambient
effects on structural health indicators**

**Identification of structural health indicators**

In this subsection, the Bayesian spectral density
approach\textsuperscript{57,58} for identifying structural health indica-
tors is briefly reviewed. This frequency-domain
approach exploits the statistical characteristics of the
discrete Fourier transform to formulate the likelihood
function of the uncertain modal parameters. The esti-
mation uncertainty of the identified parameters is quan-
tified in terms of probability distribution which reflects
the contribution of the statistical uncertainty on the
variation of the structural health indicators.\textsuperscript{59,60}
Successful applications have demonstrated its
applicability to various types of civil engineering and mechanical dynamical systems.\textsuperscript{61–63}

Let $\mathbf{\Theta}$ denote the uncertain modal parameter vector to be identified. It consists of the modal frequencies, damping ratios, mode shape components, spectral intensities of the excitation, and variance of the measurement noise. To identify the uncertain modal parameter, the power spectral density matrix of the measured structural response $\mathbf{z}$ of the monitoring time segment $[0,N_t]$ is formulated\textsuperscript{57,58}

$$
S_{z,N_t}(\omega_k) = \frac{\Delta t}{2\pi N_t} \sum_{n_i,n_i'=0}^{N_t-1} \mathbf{z}(n_i)\mathbf{z}^T(n_i') \exp[-i\omega_k(n_i - n_i')\Delta t]
$$

where $\Delta t$ is the sampling time of the measured structural response, $\omega_k = k\Delta\omega$, $k = 0, 1, \ldots, \text{INT}(N_t/2)$, $\Delta\omega = 2\pi/(N_t\Delta t)$, and $\text{INT}(\cdot)$ takes the integer part of a real number. With $N_t$ independent sets of discrete time histories $\mathbf{Z} = \{\mathbf{Z}_{s}^{(N_t)}, s = 1, 2, \ldots, N_t\}$, the spectral density matrix estimator of $\mathbf{Z}_{s}^{(N_t)}$ is denoted as $S_{z,N_t}^{(s)}(\omega_k)$ which can be computed based on the above equation. Then, the averaged spectral density matrix estimator can be obtained by $\overline{S}_{z,N_t}(\omega_k) = (1/N_t) \sum_{s=1}^{N_t} S_{z,N_t}^{(s)}(\omega_k)$. For a selected frequency range $\Xi$, the averaged spectral density matrix estimator is denoted as $\mathbf{S}_\Xi = \{\overline{S}_{z,N_t}(\omega_k), \omega_k \in \Xi\}$. By employing Bayes’ theorem\textsuperscript{57,60,64}, the posterior probability density function (PDF) of the uncertain modal parameters $\mathbf{\Theta}$ given the averaged spectral density matrix estimator $\mathbf{S}_\Xi$ is given by

$$
p(\mathbf{\Theta}|\mathbf{S}_\Xi) = \frac{p(\mathbf{\Theta})p(\mathbf{S}_\Xi|\mathbf{\Theta})}{\int_S p(\mathbf{\Theta})p(\mathbf{S}_\Xi|\mathbf{\Theta})d\mathbf{\Theta}}
$$

where $S$ denotes the entire parameter space of $\mathbf{\Theta}$. The prior PDF $p(\mathbf{\Theta})$ represents the prior knowledge of the modal parameters in $\mathbf{\Theta}$ and noninformative prior of $\mathbf{\Theta}$ is utilized in this study. The likelihood function $p(\mathbf{S}_\Xi|\mathbf{\Theta})$ is the product of Wishart distributions\textsuperscript{58,60}

$$
p(\mathbf{S}_\Xi|\mathbf{\Theta}) = \mu^{N_t/2} \prod_{\omega_k \in \Xi} |E[S_{z,N_t}(\omega_k)|\mathbf{\Theta}]|^{-N_t/2} \exp\left[-N_t\text{tr}\left\{\{E[S_{z,N_t}(\omega_k)|\mathbf{\Theta}]\}^{-1}\right\}\right]
$$

where $\mu = \pi^{-N_t/2}N_t/(N_t-N_s)! \sum_{s=1}^{N_s} (N_s - s)!$, $N_s$ is the number of monitoring channels, $E[\cdot]$ is the mathematical expectation, and $|\cdot| \text{ and } \text{tr}(\cdot)$ denote the determinant and trace of a matrix, respectively. The optimal modal parameter vector $\mathbf{\Theta}$ can then be determined by maximizing its posterior PDF

$$
\hat{\mathbf{\Theta}} = \arg \max_{\mathbf{\Theta}} p(\mathbf{\Theta}|\mathbf{S}_\Xi)
$$

The posterior PDF of the parameter vector $\mathbf{\Theta}$ can be well approximated by a Gaussian distribution with mean $\hat{\mathbf{\Theta}}$ and covariance matrix $\mathbf{S}_\Theta = [\nabla(-\ln p(\mathbf{\Theta}|\mathbf{S}_\Xi))\nabla^T|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}}]^{-1}$.\textsuperscript{57,58,60}

In this study, the identified modal frequencies are adopted as the concerned structural health indicators. The monitored ambient conditions and the identified structural health indicators are considered as the input and output of the learning network, respectively. In the following, the formulation of the learning network to describe the input–output relationship is presented.

**Development of the learning network**

The ambient conditions and the ambient-influenced structural health indicators can be expressed by the general input–output relationship

$$
\vartheta = f(\gamma) + \varepsilon
$$

where $\vartheta = \mathbf{L}_0 \hat{\mathbf{\Theta}}$ is the structural health indicator obtained from the identified modal parameters, $\mathbf{L}_0$ is the selection matrix that comprised zeros and ones, $\gamma$ is the associated monitored ambient conditions, $\varepsilon$ is the residual of the input–output relationship, and $f(\cdot)$ is the unknown mapping that reflects the contribution of the ambient conditions to the variation of the structural health indicators. The goal of this study is to construct a mapping $\tilde{f}$ based on the available data of the identified structural health indicators and the associated ambient conditions to mimic the unknown mapping $f$. Therefore, for a given set of ambient conditions $\gamma$, the structural health indicators induced by the ambient conditions can be estimated

$$
\tilde{\vartheta} = \tilde{f}(\gamma)
$$

where $\tilde{\vartheta}$ is the estimated ambient-influenced structural health indicator vector. The estimated $\tilde{\vartheta}$ provides the datum for reliable judgment on the structural health condition. By detrending the ambient effects, the variation of the structural health indicators due to the deterioration of structural integrity can be distinguished. The mapping $\tilde{f}$ is converted into an input–output relationship to be learned in a propagative manner based on the attribution of the growing training data.

Let $d_k = \{\mathbf{d}_k, \gamma_k\}$ denote the $k$th set of training data which consists of the concerned structural health indicators $\mathbf{d}_k$ and the associated ambient conditions $\gamma_k$. The collection of the first $k$ sets of training data is denoted as the training dataset $\mathcal{D}_k = \{d_1, d_2, \ldots, d_k\}$. The training output of the learning network can be expressed as

$$
\Theta_k = A_k \mathbf{W}_k
$$
where the training output $\Theta_k = \{\theta_1, \theta_2, \ldots, \theta_k\}$ consists of $k$ sets of the concerned structural health indicators, $A_k$ is the transformation feature matrix extracted from the training input $T_k = [\gamma_1, \gamma_2, \ldots, \gamma_k]$, and $W_k$ is the learning weight matrix to be determined.

To formulate the transformation feature matrix, the feature mapping groups and enhancement node groups are defined.\textsuperscript{50,51} At the beginning of the procedure, a preliminary training dataset $D_{N_P}$ with $N_P$ sets of the training data are utilized to develop the preliminary learning network. This ensures that there is sufficient information to construct the input–output relationship for the preliminary learning network. As time propagates, the effect of different selections of $N_P$ eliminates. Let $N_{F_P}$ and $N_{E_P}$ be the number of feature mapping groups and enhancement node groups of the preliminary learning network, respectively. The $f$th feature mapping group $F_f$ in the feature space basis matrix $F^{(N_{F_P})} = \{F_1, F_2, \ldots, F_{N_{F_P}}\}$ is defined as

$$F_f = F_f (\Gamma_p w_{f_f} + b_{f_f})$$

(8)

where $F_f(\cdot)$ is the $f$th feature of mapping; $w_{f_f}$ and $b_{f_f}$ are the randomly generated weight and bias of the $f$th feature of mapping, respectively. To improve the quality of the extracted information from the input data, the iterative sparse autoencoder\textsuperscript{50,65,66} is applied to convert the randomly generated weights into a set of sparse and compact features. Let $N_I$ denote the number of iteration steps. The updated weights of the $i$th iteration step can be obtained by

$$w_{f_f}^{(i+1)} = \left[\left(\Gamma_k w_{f_f}^{(0)}\right)^T \Gamma_k w_{f_f}^{(0)} + I\right]^{-1} \left[\left(\Gamma_k w_{f_f}^{(0)}\right)^T \Gamma_p + r_i - v_i\right], \quad i = 0, \ldots, N_I - 1$$

(9)

where $w_{f_f}^{(0)}$ denotes the randomly generated weights and $I$ is the identity matrix. The sparse autoencoder vectors can be updated by $w_{f_f}^{(p,q)} = \max\{|w_{f_f}^{(p,q)}|, \varepsilon|s|\}$ and $r_i = v_i + w^{(i+1)} - r_i+1$, where $|\cdot|$ is the 1-norm of a vector, $s$ is the sign function, and $\varepsilon$ is the soft thresholding coefficient. The resultant weight vector of the autoencoder $w_{f_f}^{(N_I)}$ is assigned as the tuned weights of the $f$th feature of mapping (i.e. $w_{f_f} = w_{f_f}^{(N_I)}$). To start the iteration procedure, the initial sparse autoencoder vectors are set as zero matrices, that is, $r_0 = 0$ and $v_0 = 0$.

Based on the feature space basis matrix, the enhancement node groups in the enhancement layer can be generated. The $e$th enhancement node group $E_e$ in the enhancement layer $E^{(N_{E_P})} = \{E_1, E_2, \ldots, E_{N_{E_P}}\}$ is defined as

$$E_e = E_e \left(F_e^{(N_{E_P})}w_{E_e} + b_{E_e}\right)$$

(10)

where $E_e(\cdot)$ is the nonlinear activation function; $w_{E_e}$ and $b_{E_e}$ are the randomly generated weight and bias of the $e$th enhancement node group, respectively. By utilizing the feature basis matrix and the enhancement layer given in equations (8) and (10), the transformation feature matrix $A_{N_P}$ has the form

$$A_{N_P} = \left[F_p^{(N_{F_P})}, E_p^{(N_{E_P})}\right]$$

(11)

Based on equations (7) and (11), the learning weight matrix of the prototype network can be readily obtained

$$W_{N_P} = A_{N_P}^T \Theta_P$$

(12)

where the pseudoinverse $A_{N_P}^+ = (A_{N_P}^T A_{N_P} + \delta I)^{-1} A_{N_P}^T$, with $\delta$ being the small trade-off regularization parameter for the ridge regression approximation of the pseudoinverse. Consequently, the estimated training output can be calculated by

$$\bar{\Theta}_{N_P} = A_{N_P} W_{N_P}$$

(13)

The learning network developed based on the preliminary training dataset $D_{N_P}$ is exploited as the prior network for the next time step. When a new set of training data is available, the network is updated to include the contribution of the additional training data. In the following, the formulation to obtain the updated learning network is presented.

**Propagative updating**

When a new set of training data becomes available, the learning network will be updated to adapt the new data. Let $d_{k+1} = \{\{\theta_{k+1}, \gamma_{k+1}\}\}$ denote the newly available training data. The training input and training output are then updated as $\Theta_{k+1} = \{\Theta_k, \theta_{k+1}\}$ and $T_{k+1} = \{T_k, \gamma_k\}$, respectively. Taking the additional training data into account, the updated transformation feature matrix is augmented as

$$A_{k+1} = \left[A_k \quad A_{k+1}\right]$$

(14)

where $A_{k+1} = \left[F_{k+1}^{(N_{F_P})}, E_{k+1}^{(N_{E_P})}\right]$ is an augmented submatrix to represent the extracted information of the additional input data; $F_{k+1}^{(N_{F_P})} = \{F_{N_P+1}, F_{N_P+2}, \ldots, F_{N_E}\}$ is the collection of the additional feature mapping groups to extract the information from the additional training input; and $E_{k+1}^{(N_{E_P})} = \{E_{N_E+1}, E_{N_E+2}, \ldots, E_{N_E+1}\}$ is the collection of the additional enhancement node groups induced by the additional feature mapping group. Herein, the feature mapping group
carried out to enhance the data fitting capacity of the incremental reconfiguration of the learning network is work can be incorporated efficiently. Thereafter, an equation (10) by replacing the feature space basis of the additional training data to the learning network can be expanded incrementally with the purpose of the additional enhancement node groups. The purpose of introducing enhancement node groups. The purpose of introducing

$$\mathbf{F}_f = \mathbf{F}_f (\mathbf{I}_{k+1} \mathbf{w}_f + \mathbf{b}_f)$$ \quad (15)$$

The enhancement node group \( \mathbf{E}_e \), \( e = N_{E_k} + 1, \ldots, N_{E_k+1} \), follows the expression given in equation (10) by replacing the feature space basis matrix with \( \mathbf{F}_{(k+1)}^{(EN)} \)

$$\mathbf{E}_e = \mathbf{E}_e (\mathbf{F}_{k+1}^{(EN)}) \mathbf{w}_e + \mathbf{b}_e)$$ \quad (16)$$

By applying the matrix inversion lemma, the pseudoinverse of the updated transformation feature matrix can be obtained efficiently by

$$\mathbf{A}_{k+1}^* = \mathbf{[A_k^* - B_k^* D_k^* A_k^*]}$$ \quad (17)$$

where

$$\mathbf{B}_{k+1} = \begin{cases} \mathbf{C}_{k+1}, & \text{if } \mathbf{C}_{k+1} \neq 0 \\ (\mathbf{I} + \mathbf{D}_{k+1}^T \mathbf{D}_{k+1})^{-1} \mathbf{D}_{k+1}^T \mathbf{A}_{k+1}^* & \text{otherwise} \end{cases} \quad (18)$$

$$\mathbf{C}_{k+1} = \mathbf{A}_{k+1}^* - \mathbf{A}_k^* \mathbf{D}_{k+1}$$ \quad (19)$$

$$\mathbf{D}_{k+1} = (\mathbf{A}_k^*)^T \mathbf{A}_{k+1}$$ \quad (20)$$

Based on the resultant pseudoinverse of the updated transformation feature matrix, the updated learning weight matrix can be obtained by

$$\mathbf{W}_{k+1} = \mathbf{A}_{k+1}^* \mathbf{B}_{k+1}$$ \quad (21)$$

As shown in equations (17) to (21), the pseudoinverse of the updated transformation feature matrix \( \mathbf{A}_{k+1}^* \) can be determined based on the calculated \( \mathbf{A}_k^* \) without direct computation of the pseudoinverse. It turns out that the computational cost of the updated learning weight matrix \( \mathbf{W}_{k+1} \) is low and the contribution of the additional training data to the learning network can be incorporated efficiently. Thereafter, an incremental reconfiguration of the learning network is carried out to enhance the data fitting capacity of the learning network.

**Incremental reconfiguration**

The learning network can be expanded incrementally to adapt additional feature mapping groups and/or enhancement node groups. The purpose of introducing the additional feature mapping groups is to improve the ability of the learning network in extracting the information from the training inputs. On the other hand, the purpose of the additional enhancement node group is to complexify the connection between the enhancement layer and the training output. These two implementations aim to enhance the data fitting capacity of the learning network. In general, the network architecture in deep learning is adjusted by various trials on the number of hidden layers and the number of neurons in each hidden layer, so that the resultant deep network is able to well describe the behavior of the training data. However, the computational cost can be excessive for retraining of the entire network in every trial. In contrast, the reconfiguration of the PBL network can be conducted incrementally and the procedure is computationally efficient. The formulation of the incremental reconfiguration is presented in the following.

Assume that \( N_{E_k+1} \) enhancement node groups and \( N_{Dk+1} \) feature mapping groups are introduced to the learning network with training dataset \( \mathbf{D}_{k+1} \). As a result, the updated transformation feature matrix of the reconfigured learning network can be expressed as

$$\mathbf{A}_{k+1} = \begin{bmatrix} \mathbf{A}_{k+1}^* \mathbf{F}_{(k+1)}^{(N_{E_k+1})} \mathbf{E}_{\mathbf{e}_{k+1}}^{(N_{D_k+1})} \mathbf{E}_{\mathbf{f}_{k+1}}^{(N_{D_k+1})} \end{bmatrix}$$ \quad (22)$$

where \( \mathbf{F}_{(k+1)}^{(N_{E_k+1})} \) is the submatrix associated with the additional feature mapping groups, \( \mathbf{E}_{\mathbf{e}_{k+1}}^{(N_{D_k+1})} \) is the submatrix associated with the additional enhancement node groups, and \( \mathbf{E}_{\mathbf{f}_{k+1}}^{(N_{D_k+1})} \) is the enhancement node groups induced by the additional feature mapping groups. These three submatrices are given by

$$\mathbf{F}_{(k+1)}^{(N_{E_k+1})} = \begin{bmatrix} \mathbf{F}_{N_{E_k+1}+1}, \ldots, \mathbf{F}_{N_{E_k+1}+N_{D_k+1}} \end{bmatrix}$$ \quad (23)$$

$$\mathbf{E}_{\mathbf{e}_{k+1}}^{(N_{D_k+1})} = \begin{bmatrix} \mathbf{E}_{N_{E_k+1}+1}, \ldots, \mathbf{E}_{N_{E_k+1}+N_{D_k+1}} \end{bmatrix}$$ \quad (24)$$

$$\mathbf{E}_{\mathbf{f}_{k+1}}^{(N_{D_k+1})} = \begin{bmatrix} \mathbf{E}_{N_{E_k+1}+N_{D_k+1}+1}, \ldots, \mathbf{E}_{N_{E_k+1}+N_{D_k+1}+N_{D_k+1}} \end{bmatrix}$$ \quad (25)$$

where \( \mathbf{F}_f, f = N_{F_k+1} + 1, \ldots, N_{F_k+1} + N_{D_k+1} \), follows the same expression given in equation (15) and \( \mathbf{E}_e, e = N_{E_k+1} + 1, \ldots, N_{E_k+1} + N_{D_k+1} \), follows the same expression given in equation (16). For the enhancement node groups induced by the additional feature mapping groups, they can be obtained by replacing the feature space basis matrix in equation (16) with \( \mathbf{F}_{(k+1)}^{(N_{E_k+1})} \), that is, \( \mathbf{E}_e = \mathbf{E}_e (\mathbf{F}_{(k+1)}^{(N_{E_k+1})} \mathbf{w}_e + \mathbf{b}_e) \), \( e = N_{E_k+1} + 1, \ldots, N_{E_k+1} + N_{D_k+1} \).

By applying the matrix inversion lemma, the pseudoinverse of the modified transformation feature matrix can be obtained by

$$\mathbf{(A}_{k+1}^* \mathbf{B}_{k+1})^+ = \mathbf{[A}_{k+1}^* - \mathbf{D}_{k+1} \mathbf{B}_{k+1}]$$ \quad (26)$$
where

\[
B_{k+1} = \begin{cases} \frac{C_{k+1}}{1 + \lambda(D'_{k+1})^T D_{k+1}^*} & \text{if } C_{k+1} \neq 0 \\ I + (D_{k+1}^*)^{-1}(D'_{k+1})^T A_{k+1}' \end{cases} \tag{27}
\]

\[
C_{k+1} = [F_{T_{k+1}}(N_{k+1}), E_{E_{k+1}}(N_{k+1}), E_{E_{k+1}}(N_{k+1})] - A_{k+1}D_{k+1} \tag{28}
\]

\[
D'_{k+1} = A'_{k+1} [E_{F_{k+1}}(N_{k+1}), E_{E_{k+1}}(N_{k+1}), E_{E_{k+1}}(N_{k+1})] \tag{29}
\]

Thereafter, the updated learning weight matrix of the reconfigured network can be readily obtained

\[
W'_{k+1} = (A'_{k+1})^* \Theta_{k+1} \tag{30}
\]

Via equations (26) to (30), the pseudoinverse \((A'_{k+1})^*\) can be obtained efficiently based on the inherent results from the trained network. Then, the training output of the learning network can be obtained by multiplying the updated transformation feature matrix and the updated learning weight matrix of the reconfigured network

\[
\hat{\Theta}_{k+1} = A'_{k+1} W'_{k+1} \tag{31}
\]

Furthermore, the estimation performance of the learning network is evaluated by the root mean square error (RMSE)

\[
RMSE_{k+1} = \frac{1}{k+1} \sum_{i=1}^{k+1} \left\| \hat{\Theta}_i - \hat{\Theta}_i \right\|_2 \tag{32}
\]

The reconfiguration procedure will be terminated if additional feature mapping groups and enhancement node groups cannot provide considerable improvement on the data fitting capacity of the learning network. Herein, if the reconfiguration with an additional feature mapping group and an enhancement node group provides less than 1% of further reduction on the resultant RMSE or the numbers of the additional mapping groups and enhancement node groups reach the threshold numbers (i.e. \(N_{F_E} = N_{F_E}\) and \(N_{D_E} = N_{D_E}\)), the reconfiguration procedure will be terminated. A lower tolerance percentage provides less flexible termination criteria, which may require more feature mapping groups and/or enhancement node groups in the learning network, and vice versa. Since the improvement of the overall performance, the tolerance percentage is set at 1% in this study. The resultant trained network will be used to estimate the structural health indicators for the given ambient conditions and it will be served as the prior learning network for the next time step.

**Summary of the proposed approach**

The flowchart of the proposed approach is shown in Figure 1 and the schematic plots of the learning networks in different steps are shown in Figure 2. The procedure is summarized in as follows:

1. Construct the preliminary learning network (Figure 2(a))
   (i) Set \(N_F\);
   (ii) Identify the optimal modal parameters using equation (4);
   (iii) Form the preliminary training dataset \(D_{NP}\);
   (iv) Set \(N_{FP} + N_{FP}\);
   (v) Compute \(F_{NP}^{(NP)}\) and \(E_{NP}^{(NP)}\) using equations (8) and (10), respectively;
   (vi) Compute \(A_{NP}\) using equation (11);
   (vii) Compute \(W_{NP}\) using equation (12);
   (viii) Compute \(\Theta_{NP}\) using equation (13);

2. For the \((k+1)\)th time step, expand the network to adapt the additional set of training data (Figure 2(b))
   (i) Identify the optimal modal parameters using equation (4);
   (ii) Form the \((k+1)\)th set training data \(D_{k+1}\);
   (iii) Set \(N_{NP}\);
   (iv) Compute \(F_{k+1}^{(NP)}\) and \(E_{k+1}^{(NP)}\) based on equations (15) and (16), respectively;
   (v) Compute \(A_{k+1}^{(NP)}\) using equation (17);
   (vi) Compute \(W_{k+1}^{(NP)}\) using equation (21);

3. Modify the network to improve the data fitting capacity (Figure 2(c))
   (i) Set \(N_{NP}\); and \(N_{DP}\);
   (ii) Compute \(F_{k+1}^{(NP)}\), \(E_{k+1}^{(NP)}\), and \(E_{k+1}^{(NP)}\) given in equations (23) to (25), respectively;
   (iii) Compute \((A_{k+1})^*\) using equation (26);
   (iv) Compute \(W_{k+1}^{(NP)}\) using equation (30);
   (v) Compute \(\Theta_{k+1}^{(NP)}\) using equation (31);
   (vi) Calculate \(RMSE_{k+1}\) using equation (32);

4. Check the termination criteria. If the criteria are not satisfied, repeat step (3). Otherwise, the resultant network of the current time step is settled and go to step (5).

5. Use the resultant network for estimation and go to step (2) for the next time step.

**Case study on long-term structural health monitoring**

*Monitored structure and instrumentation*  

In this study, the long-term in-field monitoring measurement of a residential building is analyzed. The
monitored structure is a 22-story reinforced concrete building inaugurated in 2005 for athletes lodging in the Fourth East Asian Games hosted in Macau. The height of the building is 64.70 m with an L-shaped floor plan. The lengths of the two spans are 51.90 and 61.75 m, respectively. Figure 3 shows the side view and a typical floor plan of the monitored building. The monitoring system included accelerometers, thermometers, and an anemometer to measure the structural response as well as the operating ambient conditions. In particular, two geophone spring–mass type accelerometers were placed perpendicularly along the two spans at the junction (indicated with the two measurement directions shown in Figure 3(b)) to capture the structural acceleration response. The sampling frequency was 200 Hz with a sensitivity of 50 V/g. Wireless thermometers were mounted to record the ambient temperature and relative humidity. The measurement resolution of temperature and relative humidity were 0.1°C and 1%, respectively. Wind velocities of the two horizontal directions were measured by the anemometer mounted on the top of the building with a resolution of 0.01 m/s.

Operating ambient conditions and identified structural health indicators

Three years of continuous monitoring, from 1 July 2011 to 30 June 2014, was conducted. To minimize the spatial temperature variation of the building, daily measurements recorded at 23:00 for 10 min are examined. Hence, the database contained 1096 sets of acceleration time histories and the operating conditions, including the averaged operating temperature \( T \), relative humidity \( H \), and wind velocities \( U \) and \( V \). It is noted that the positive direction of the wind velocities \( U \) and \( V \) refers to the East and North directions, respectively. The statistical properties of the four ambient conditions are summarized in Table 1. During the 3-year monitoring period, 15 tropical cyclones centered within 800 km of Macau. Five of them developed into severe typhoons, which induced wind speeds of over 62 km/h (i.e. a gale of category no. 8 in the Beaufort scale) to the region. Figure 4 shows the time histories of the monitored ambient conditions. The monitoring periods under the five severe typhoons are highlighted and the periods under the 10 low-impact tropical cyclones are shaded. The time histories of the ambient conditions demonstrated notable seasonal variation. For example, the temperature values measured during the summers (June to September) were relatively high and less fluctuated. The measurements of relative humidity recorded during the dry seasons (December to January) were the most fluctuated throughout the year. For the wind velocities, dramatic changes were observed under the severe typhoon-affected periods.
Besides the ambient conditions, the acceleration response of the building throughout the entire monitoring period was recorded. Figure 5 shows two typical sets of acceleration responses monitored under calm wind condition (measured on 1 July 2011) and severe typhoon condition (measured on 23 July 2013). It is observed that the acceleration response of the building was significantly higher under severe typhoon. For both directions, the measurements under severe wind condition were around five times higher than those under calm wind condition. By utilizing the measured structural response, the Bayesian spectral density approach was applied to determine the structural health indicators and the associated estimation uncertainty. In this study, we focus on modeling of the ambient interferences on the modal frequencies. The time histories of the identified modal frequencies with the plus and minus three standard deviations credibility intervals of the first two modes are shown in Figure 6. Again, the monitoring periods under the five severe typhoons are highlighted and the periods under the 10 low-impact tropical cyclones are shaded. The identified modal frequencies exhibited obvious seasonal cycles during the 3 years of monitoring. However, there was no evidence of structural damage since the changes of the modal frequencies were recovered to the original
levels after the 3 years of monitoring. By taking into consideration the seasonal variation, the credibility intervals of the modal frequencies are sufficiently narrow and it confirms that the various ambient conditions induced considerable influences on the changes of the modal frequencies. For the two modes of concern, the identified modal frequency of the first mode is in the range of $\frac{1}{35}$, $\frac{1}{45}$ Hz and that of the second mode is in the range of $\frac{1}{58}$, $\frac{1}{70}$ Hz. The difference between the maximum and minimum values is over 7% in both modes. Moreover, severe wind loading induced sudden drops on all the modal frequencies and the reductions were recovered after the effects of the severe typhoon had vanished.

Figure 7 shows the scattered plots of the concerned ambient conditions versus the identified modal frequencies. Over the 3-year monitoring period, the structure experienced various ambient conditions and the relationships between the ambient conditions and the modal frequencies were complicated. For example, regarding the temperature effect, it is found that the data points of the two modes had increasing trends when the temperature increased. For the wind velocity, decreasing trends of the modal frequencies and wind velocity in the $V$ direction were observed. Since the relationships between the ambient conditions and the structural health indicators are complicated, it is challenging to construct an appropriate parametric formulation for describing their relationships. In the following, the proposed PBL is implemented for data-driven nonparametric modeling of these relationships.

Table 1. Statistical properties of the ambient conditions.

<table>
<thead>
<tr>
<th></th>
<th>$T$ (°C)</th>
<th>$H$ (%)</th>
<th>$U$ (m/s)</th>
<th>$V$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>26.6</td>
<td>84.7</td>
<td>2.35</td>
<td>0.41</td>
</tr>
<tr>
<td>Minimum</td>
<td>11.0</td>
<td>36.0</td>
<td>-8.69</td>
<td>-11.9</td>
</tr>
<tr>
<td>Maximum</td>
<td>35.7</td>
<td>99.0</td>
<td>26.5</td>
<td>15.8</td>
</tr>
</tbody>
</table>

To conduct the nonparametric modeling, the ambient conditions were normalized to dimensionless variables with similar scales. In particular, the temperature and wind velocity measurements were normalized with $\bar{a} = (a - a_{lb})/(a_{ub} - a_{lb})$ where $a$ is the raw data of the monitored ambient variable and $a_{ub}$ and $a_{lb}$ are the upper and lower bounds of the corresponding monitored quantity for the normalization, respectively. The upper and lower bounds were selected to cover the historical maximum and minimum values of the ambient condition. Herein, $a_{ub} = 50^\circ$C and $a_{lb} = 0^\circ$C were used for temperature measurements, and $a_{ub} = -30$ m/s and $a_{lb} = 30$ m/s were used for wind velocity measurements. As a result, all the training inputs fall in the range of $[0, 1]$. The data of the first 2 years (1 July 2011 to 30 June 2013) with 731 sets of training data were utilized to develop the learning network of PBL. Herein, the preliminary learning network was established using the first 10 sets of data (i.e. $N_p = 10$) and the learning network was updated adaptively for every new set of training data afterward. Then, the performance of the resultant learning network was validated based on the data captured in the third year (1 July 2013 to 30 June 2014). In other words, the last 365 sets of data were used to evaluate the estimation performance of the...
learning network, but they were not involved in the training process. The preliminary learning network starts with a simple architecture with two feature mapping groups and two enhancement node groups (i.e. $N_{FP} = 2$ and $N_{EP} = 2$).

The assigned quantities to develop the learning network of the PBL are summarized in Table 2. By implementing the proposed approach, the ambient influences on the concerned structural health indicators are modeled to provide the baseline values of the indicators under the given ambient conditions. Figure 8 shows the estimated modal frequencies obtained from the resultant learning network and the identified results. The solid and dashed lines refer to the estimated and identified results, respectively. The subplots on the two rows refer to the results of the two concerned modes. The subplots on the left column are the one-step ahead estimation, while the subplots on the right column are the estimation based on the resultant learning network trained with the data in the first 2 years. Moreover, the corresponding ratio of residual between the estimated and identified results is shown in Figure 9. The results show that the learning network achieves satisfactory performance in capturing the ambient influences and the estimation can trace the seasonal variation of the identified modal frequencies. The difference between the estimated and identified results is less than 3% in both the one-step ahead estimation and the validation estimation.

Figure 10 shows the estimated modal frequencies versus the identified values, and 45° lines of perfect match are drawn for reference. Again, the subplots on the two rows refer to the results of the two concerned modes. The subplots on the left column are the one-step ahead estimation, while the subplots on the right column are the estimation based on the resultant learning network.
Figure 5. Acceleration time histories: (a) calm wind condition (direction 1), (b) severe wind condition (direction 1), (c) calm wind condition (direction 2), and (d) severe wind condition (direction 2).

Figure 6. Time histories of the identified modal frequencies and the associated 99.7% credibility intervals.
learning network trained with the first 2 years of data. It is found that the data points generally distribute around the lines of perfect match which indicates that the resultant learning network successfully models the ambient inferences on the modal frequencies without statistical bias. The resultant relationship serves as the Figure 7. Scatter plots of the ambient conditions and the identified modal frequencies.

Table 2. Settings of the learning network.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$N_I$</th>
<th>$\kappa$</th>
<th>$\delta$</th>
<th>$N_P$</th>
<th>$N_{FP}$</th>
<th>$N_{EP}$</th>
<th>$N_{AE}$</th>
<th>$N_{AF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assigned value</td>
<td>50</td>
<td>$10^{-3}$</td>
<td>$2^{-30}$</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 8. Estimated and identified modal frequencies: (a, c) one-step ahead estimation of the two modes, and (b, d) estimation based on trained network of the two modes.

Figure 9. Residual of the estimated modal frequencies: (a) first mode, and (b) second mode.
baseline reference to detrend the ambient interference on the variation of the modal frequencies.

Figure 11 shows the design profiles of the modal frequencies under varying ambient conditions calculated based on the resultant learning network. The monitored ambient conditions of the first 2 years are depicted with circles and the data of the last year are depicted with crosses. The sizes of the markers vary linearly with the value of the identified modal frequencies. The subplots on the upper row depict the relationship under varying temperature and relative humidity with zero wind velocity in both $U$ and $V$ directions. The subplots on the bottom row are the estimated modal frequencies under varying wind velocity with the fixed temperature at 25°C and relatively humidity at 85%. Under zero wind velocity (Figure 11(a) and (b)), the minimum design modal frequencies occur when both temperature and relative humidity are low. The design values increase as the temperature and relative humidity increase. Under the fixed temperature and relative humidity (Figure 11(c) and (d)), the design values are relatively higher when the wind velocities of both directions are within the negative regime (i.e. $[-10, 0]$m/s) and the design values decrease as the wind velocity increases. The results show that the design profiles can match well the values of the identified modal frequencies. The design profiles provide a convenient illustration of the variation of the modal frequencies under varying ambient conditions.

**Comparison with parametric modeling**

For comparison, parametric modeling with Bayesian estimation is formulated to describe the ambient-influenced structural health indicator model. With the same training and validation information as the resultant learning network, the data of the first 2 years (1 July 2011 to 30 June 2013) were utilized to train the parametric model, while the data of the third year (1 July 2013 to 30 June 2014) were utilized to examine the estimation performance of the resultant model. The monitored temperature and wind velocity were normalized with the aforementioned normalization so that all the ambient data fall in the range of $[0, 1]$. We assume that the parametric model consists of the constant, linear components of each of the ambient conditions, and two intercorrelated terms between temperature and relative humidity, and the wind velocities of the two directions. Therefore, the relationship between the ambient influenced and the modal frequency of the $m$th mode is given by
\[ \hat{\Theta}_m(\beta; b_m) = \sum_{b=1}^{N_B} b_m \beta_b \]  

where \( \beta = [1, \bar{T}, \bar{H}, \bar{T}H, \bar{U}, \bar{V}, \bar{U}V] \) is the normalized ambient condition component vector, \( b_m = [b_{m1}, \ldots, b_{m7}] \) is the uncertain model parameter vector, and \( \Theta = [\bar{\omega}_1, \bar{\omega}_2] \). For the model output error of the \( m \)th mode, that is, the difference between the identified and estimated modal frequencies, they are assumed to be independent Gaussian random variables with zero mean and identical variance \( \sigma_m^2 \). Based on Bayes’ theorem, the posterior PDF of the uncertain parameters, given the training data \( D_n \), can be expressed as

\[
p(b_m, \sigma_m | D_n) \propto p(b_m, \sigma_m) p(D_n | b_m, \sigma_m)
\]

where \( p(b_m, \sigma_m) \) is the prior PDF of \( b_m \). A noninformative prior is applied in this study, that is, the prior PDF \( p(b_m, \sigma_m) \) follows a uniform distribution. The likelihood function \( p(D_n | b_m, \sigma_m) \) has the form

\[
p(D_n | b_m, \sigma_m) = (2\pi)^{-\frac{N_n}{2}} \sigma_m^{-N_n} \exp \left\{ -\frac{1}{2\sigma_m^2} \sum_{k=1}^{N_n} \left[ \bar{\omega}_m - \hat{\Theta}_m(\beta_k; b_m) \right]^2 \right\}
\]

where \( N_n \) is the number of training data in \( D_n \), \( \bar{\omega}_m = \omega_{m\bar{k}} \) is the identified modal frequency of the \( m \)th mode at the \( k \)th time step, and \( \beta_k = [1, \bar{T}_k, \bar{H}_k, \bar{T}_k\bar{H}_k, \bar{U}_k, \bar{V}_k, \bar{U}_k\bar{V}_k] \) is the normalized ambient conditions monitored at the correspondent time step. By maximizing the posterior PDF, the optimum uncertain model parameters have a closed-form solution given by Yuen and Kuok

\[
\hat{b}_m = B^{-1} z_m
\]

where the \((i,j)\)th component of the matrix \( B \) is given by

\[
B^{(i,j)} = \sum_{k=1}^{N_B} \beta_{ki} \beta_{kj}, \quad i, j = 1, \ldots, N_B
\]

and the \( i \)th component of the vector \( z_m \) is given by

\[
z_m^{(i)} = \sum_{k=1}^{N_B} \bar{\omega}_m \beta_{ki}
\]

By utilizing the training data, the resultant model of the modal frequencies has the form.
The resultant parametric model shows that the signs of the coefficients associated with the same ambient conditions are identical in the two modes. The estimation performance of these resultant parametric models was evaluated based on the testing data recorded in the third year. To compare the estimation performance of the resultant parametric models and the learning network, Figure 12 shows the estimated results of the testing points versus the identified values with the lines of perfect match. The results show that the estimation of the parametric models suffers from considerable bias. For the first modal frequency, most of the data points are distributed on the upper side of the line of perfect match when the identified values are lower than $v_1 = 1.4$ Hz. Moreover, most of the data points are distributed on the lower side of the line of perfect match when the modal frequencies were $\omega_1 > 1.4$ Hz. It indicates that the estimated values are generally higher than the target values when $\omega_1 < 1.4$ Hz but the estimated values are generally lower than the identified values when $\omega_1 > 1.4$ Hz. For the modal frequencies of the second mode, the estimated values are generally higher than the target values. By comparing the results obtained from the parametric models and the learning network (i.e. Figure 12(a) with Figure 10(b) and Figure 12(b) with Figure 10(d)), the results show that the learning network achieves superior performance in describing the ambient influences on the concerned structural health indicators than the parametric models.

\[
\begin{bmatrix}
    2.244 & 0.003 & -0.125 & 0.376 & -0.583 & -0.440 & 0.601 \\
    2.927 & 0.042 & -0.028 & 0.269 & -0.459 & -0.459 & 0.567
\end{bmatrix} \beta
\]

The resultant parametric model shows that the signs of the coefficients associated with the same ambient conditions are identical in the two modes. The estimation performance of these resultant parametric models was evaluated based on the testing data recorded in the third year. To compare the estimation performance of the resultant parametric models and the learning network, Figure 12 shows the estimated results of the testing points versus the identified values with the lines of perfect match. The results show that the estimation of the parametric models suffers from considerable bias. For the first modal frequency, most of the data points are distributed on the upper side of the line of perfect match when the identified values are lower than $v_1 = 1.4$ Hz. Moreover, most of the data points are distributed on the lower side of the line of perfect match when the modal frequencies were $\omega_1 > 1.4$ Hz. It indicates that the estimated values are generally higher than the target values when $\omega_1 < 1.4$ Hz but the estimated values are generally lower than the identified values when $\omega_1 > 1.4$ Hz. For the modal frequencies of the second mode, the estimated values are generally higher than the target values. By comparing the results obtained from the parametric models and the learning network (i.e. Figure 12(a) with Figure 10(b) and Figure 12(b) with Figure 10(d)), the results show that the learning network achieves superior performance in describing the ambient influences on the concerned structural health indicators than the parametric models.

**Conclusion**

In this article, PBL is proposed to model the relationship between ambient conditions and structural health indicators. The proposed approach provides an effective model-free, data-driven tool for nonparametric modeling with growing data. To handle the growing data generated in long-term structural health monitoring, the learning network of PBL is updated to adapt the newly available data as time progresses. Instead of retraining the entire learning network, the updating is carried out progressively to include the contribution of the additional training data based on the trained network of the previous time step. Moreover, the learning network can be reconfigured incrementally to enhance the data fitting capacity. The expandable architecture allows the learning network to be updated and reconfigured efficiently. The proposed approach was applied to develop the ambient-influenced structural health indicator model. The measurements of 3-year full-scale continuous monitoring on a residential reinforced concrete building were utilized for demonstration. The results showed that the resultant learning network successfully modeled the ambient effects on the structural health indicators and provided satisfactory estimation on the structural health indicators for the given ambient conditions. By detrending the ambient interference on the variation of the structural health indicators, the resultant relationship serves as the baseline reference for reliable structural damage assessment.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or
publication of this article: This research has been supported by the Science and Technology Development Fund of the Macau SAR under research grant SKL-IOTSC-2018-2020 and by the Research Committee of University of Macau under research grant CPG2019-00023-FST. These generous supports are gratefully acknowledged.

ORCID iDs
Sin-Chi Kuok https://orcid.org/0000-0001-7363-6761
Ka-Veng Yuen https://orcid.org/0000-0002-1755-6668

References


64. Beck JL. Bayesian system identification based on probability logic. Struct Control Hlth 2010; 17: 825–847.