Introducing the HFTE Model: A Multi-Species Predator–Prey Ecosystem for High-Frequency Quantitative Financial Strategies

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Abstract
In this article we take a new approach to studying electronic trading and systemic risk by introducing the HFTE model. We specify an approach in which agents interact through a topological structure designed to address the complexity demands of most common high-frequency strategies but designed randomly at inception. The primitive strategy ecosystem is then studied through a simplified genetic algorithm. The results open up intriguing social and regulatory implications with the helping doors of mathematical biology and game theory, which specific mirror points have been summarized for the sake of illustrating the puzzling findings.

Keywords
HFTE model, high-frequency financial funnel (HFFF), multi-target tracking, stability of financial systems, data analysis and patterns in data, electronic trading, systemic risk, high-frequency trading, game theory, machine learning, predator–prey models

1 Introduction

1.1 Historical context
After the subprime crisis of 2008 and the resulting social uproar, governments strongly pushed the regulators to develop more efficient risk-monitoring systems.1 Given that the biological ramifications of the unfortunate cost of pattern recognition [15] are inherent to humans and the fact that the historical crises were not directly connected to each other, it became implicitly clear that the next financial crisis could not be in real estate again (at least not immediately...) but rather elsewhere. The candidate sector under coercion became very quickly that of algorithmic systematic trading, the most famous incident of which was the flash crash of May 6, 2010, in which the Dow Jones Industrial Average lost almost 10 percent of its value in a matter of minutes. However, the current state-of-the-art risk models are the ones inspired by the last subprime crisis and are essentially models of networks in which each node can be impacted by the connected nodes through contagion [10], perhaps better suited for lower-frequency models. Indeed, on June 8, 2011, a seemingly relatively unnoticed event occurred on the natural gas commodities market. We mention here “relatively unnoticed” simply because the monetary impact was limited and finance is unfortunately an industry in which warning signs are usually dismissed until it is too late. We can see from Figure 1 that clearly something nonrandom is occurring. This feeling is exacerbated with the strong intuition that only interacting agents falling into some sort of quagmire could yield such fascinating series of increasing oscillations followed by a mini crash. Until the arrival of the algorithm, never in more than 100 years of data in countless products at different geographical locations was anything as clear in terms of oscillation observed, and this observation came in an immature market for electronic trading. I would like to spend a few lines developing this point, as it may not be clear to the reader. Indeed, commodities have historically been seen as a physical market, this in turn meaning that the prices are driven by supply and demand of commodities which can be consumed, stored, and/or produced. This particular point is a unique feature compared to the other markets (equities, FX, or rate). Also, Figure 1 suggests that the common, though perhaps a bit lazy, view that crashes occur through totally unpredictable [30] events may not be true for algorithmic trading.

1.2 Scientific method and parallel to Conway’s Game of Life
In this article we take an approach similar in methodology to Conway’s Game of Life [8], a four-rule cellular automaton exercise for which Figure 2 reminds us of the

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1 In this context risk is viewed as a mixture of market and reputational risk.
rules and Figure 3 provides a three-picture snapshot of one random simulation. We apply Conway’s methodology to the world of high frequency trading (HFT) while adjusting some of the idiosyncratic parts of the exercise. As a reminder, Conway’s Game of Life assumes that complexity in an ecosystem arises from simple rules. For instance, these rules can lead to a family of three different types of automata (when iterations are increased and the seed is random). As a reminder, you may get:

- Stable forms (for example the ‘Block’, the ‘Beehive’, the ‘Loaf’, the ‘Boat.’) Intuitively the reader may guess that the concept of financial stability may be raised through a similar methodology.
- Oscillating forms (for example the “Blinker,” the “Toad,” the “Beacon,” the “Pulsar,” the “Pentadecathlon.”) Intuitively the reader may guess that the concept of financial cycles or HF oscillations as in Figure 1 may be induced through a similar scientific methodology.
- Moving forms (for example the “Glider” and the “LWSS” lightweight spaceship) which may have different sizes and speeds.

The parallel to the world of quantitative financial strategies would be the following few points:

- Interacting agents lead to the market price fluctuations and more specifically their sole interaction determines the stability or instability of the market depending on what the market is made of in terms of the strategies involved as well as the evolving order book.
- The market will follow the rules of a zero-player game with, however, random seeds.

\[ p_i^t = m_i \left( 1 + (-1 \times 1_{i1} + 1 \times 1_{i0}) \times 0.001\% \right) \]

Remark Figure 4 represents an order book as described by the previous definition.
Recently the concept of an ecosystem of strategies [12] was introduced. Though the idea has great potential, the paper assumes a set of static strategies which do offer to some extent an interesting current snapshot of the market but do not offer:

- a history for this snapshot;
- an inspiring future for the field;
- a topology for these strategies (in the form of a DNA) on which one could study the complex problem in another mathematical domain, easier to solve;
- a sense of how to study the stability of the markets as suggested by the term “ecosystem” and its biological meaning;
- social insight about how this should impact the regulatory horizon;
- a connection to other fields in which mirror concepts and properties could be used to increase our mathematical weaponry in the context of analyzing critical concepts such as stability or cycles.

**Definition** We call HFTE the high-frequency trading ecosystem Ecosystem model which attempts to answer the six bullet points just raised, the subject of this article.

**Remark** The naming of the model proved a bit challenging. Combinations of the following phrases were assessed:

- high-frequency trading;
- quantitative strategies;
- multi-species predator–prey ecosystem;
- financial automata.

Ultimately HFTE prevailed due to its connection to HFT, which almost everyone in the financial industry knows about, and “E” (for ecosystem), which really is the key idea from the article.

### 1.4 Problem formulation and agenda

#### 1.4.1 Problem formulation

The connection between machine learning and HFT has long been implicitly established via the numerous systematic trading positions available in most job-searching tools (eFinancialCareers, LinkedIn, etc.). It is unclear, however, which of the numerous machine-learning techniques is most relevant to what high-frequency traders wish to accomplish. The field of machine learning itself is quite rich; genetic algorithm, algorithmic game theory, state-space models, Kalman filter, sequential Monte Carlo methods, support vector machine, neural networks, or even a simple multi-linear regression are some of the keywords mentioned in job descriptions whose title would not suggest much difference in the tools used by the quants supposed to perform the tasks associated with these jobs. However, what most of these methodologies have in common is that they assume a pattern inherent to the market itself as opposed to taking the market as a consequence of the strategies composing this market.  

**Remark** An interesting analogy can be made with respect to how the gene-centered view of evolution (as opposed to the individual-centered view of evolution) completely re-shuffled our understanding of natural selection and gave the opportunity to see altruism at a different enhanced angle. By analogy we are trying to communicate the idea that the market-centered view of the financial system is the wrong way around for understanding the fluctuation of the market and that the strategy-centered view of the financial system provides the opportunity to look at the market from a different enhanced angle.

#### 1.4.2 Agenda

We will first introduce in Section 2 a generalized network topology which we speculate as having enough architectural DNA to have the potential to formalize most classic financial strategies, for which we will give a few examples. In Section 3 we will specify our genetic algorithm (GA) as a mean to study the high-frequency market, which essentially, with Section 2, is the core mathematical engine (the HFTE model) to help us keep track of the various strategy families’ performance in our environment through time. We will analyze in Section 4 the results and will provide on that occasion a parallel to the world of mathematical biology, more specifically in Section 4.3 its connection to predator–prey models and in Section 4.2 its link to some of the interesting results in game theory, namely evolutionary dynamics. We will further expand on our findings by providing a couple of applications in Section 5.1, more specifically in HFT and in high-level regulatory and government policies. Finally in Section 6 we will conclude our article by suggesting potential continuation for research in the HFTE model.

**Remark** This article and its second part are at the crossroads of a few different fields; Figure 5 attempts to guide the reader in these fields for preparatory sake. However, the article has been written in such a way as to be accessible to the biggest possible audience, including practitioners, at the risk of lacking a bit of rigor from time to time.

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1. Top-down vs. bottom-up approach.

2. Geometry.

3. For example, game theory, mathematical biology, signal processing.

4. The particle filter part is currently being ironed out.
Figure 5: Academic schemes and fields involved in this article and its follow up

2 Network topology and classic financial strategies

2.1 Network topology and learning potential

Two important milestones in machine learning are worth remembering, as they shed light on why the core building blocks of our HFTEmodel are a certain way. First, Warren McCulloch and Walter Pitts [23] introduced their threshold logic model in 1943, which is agreed to have guided the research in network topology in artificial intelligence for more or less a decade. Second, Rosenblatt [25] formally introduced the perceptron concept in 1962 (though some early-stage work had started in the 1950s). The idea of the perceptron was one in which the inputs \( x_1 \) and \( x_2 \) as depicted in Figure 6 could act as separators [15] and therefore a direct equivalence could be made to the multi-linear regression which we will elaborate on in more detail is Section 3.2.2.

One observed limitation of the perceptron as described by Rosenblatt, in 1969, was that a simple yet critical well-known function such as the XOR function could not be modeled [19]. This resulted in a loss of interest in the field until it was shown that a feedforward artificial neural network (ANN) with two or more layers could in fact model these functions (see Figure 7 for the illustration). Added to this we have the well-known overfitting [29] problem when it comes to supervised learning, which has been there since inception, though regular progress is being made in that domain without any real breakthrough.

2.2 The funnel

These few historical rationals are the main drivers which have led us to propose the funnel, introduced by Martin Nowak [21], as the simplest possible network to model (therefore minimizing overfitting) the key functions for our application. The area of evolutionary graph theory is quite rich. Many graphs provide interesting properties. We can formalize the learning process from all of our strategies using the topology of Figure 8 by providing a set \( \mathcal{T} \), as described by equation (2), of weights corresponding to all the possible weights of this particular figure:

\[
\mathcal{T} = \left\{ \frac{\sum_{i=1}^{9} w_{ij}}{\sum_{i=1}^{9} w_{ij}} \right\}
\]

with \( w^i, w^h, \) and \( w^o \), respectively, the weights associated with the input, hidden, and output layers.
Remark Note that in the context of this article we have chosen to work with Martin Nowak's [21] funnel, whose modification is described in Figure 8. This topological structure offers the advantage of linking some interesting bridges between the worlds of

- information theory, since it also resembles the classic structure of a neural network and can therefore easily accommodate the mapping of classic and less classic financial strategies,
- evolutionary dynamics since Moran-like processes can easily be formalized, and biologically, since it is a potent amplifier of selection [21].

We conclude this subsection by providing a definition of the HFFF below.

Definition The HFFF is a topological structure of nine inputs, three hidden layers, and one output layer. Each node connects to the next layer and to itself. Each connection to itself will be labeled by $w_j$ and the others by $w_i$. We admit that $w_j \sim U[-1, 1]$ and that $w_i \sim U[0, 1]$, and therefore the results from equation (3):

$$w_j \sim U[-\lambda, 1]$$

(3)

2.2.1 The trend-following topology

A very common trading strategy is trend following (TF). The idea of TF is that if the price has been going a certain way (e.g., up or down) in the recent past, then it is more likely to follow the same trend in the immediate future.

Definition The mathematical formulation of TF can be diverse, but in the context of this article we will be using an exponentially weighted moving average (EWMA) formally described by equation (4):

$$\hat{x}_t = (1 - \lambda)x_t + \lambda\hat{x}_{t-1}, \lambda \in [0, 1]$$

(4)

In this equation $\lambda$ represents the smoothness parameter with $\lambda \in [0, 1]$.

Figure 8: The high-frequency financial funnel

![Image of a funnel diagram](image)

$$\ldots + \Delta b_5 + \Delta b_4 + \Delta b_3 + \Delta b_2 + \Delta b_1 + \Delta m + \Delta a_2 + \Delta a_1 + \Delta a_0 + \Delta a_5 + \ldots$$

Remark The lower the $\lambda$, the more the next move will be conditional on the immediately adjacent previous move. Conversely, the higher the $\lambda$, the more the future move will be a function of the long-term trend. The idea is that through a simple filtering process, the noise is extracted from the signal, which then returns a clean time series $\hat{x}_t$. Traders seldom like to use directly, or sometimes use with a couple of other similar equations with a different $\lambda$ and therefore define a signal as a difference of these various filtered time series.

Proposition The HFFF can model TF strategies.

Proof. Simply set $U_{[1,4]}w_{ij}^0 = 0, U_{[6,9]}w_{ij}^0 = 0, U_{[6,9]}w_{ij}^1 = 0, U_{[6,9]}w_{ij}^2 = 0, U_{[1,4]}w_{ij}^3 = 0, U_{[1,4]}w_{ij}^4 = 0, U_{[1,4]}w_{ij}^5 = 0, U_{[1,4]}w_{ij}^6 = 0, w_{0,j}^0 = 0, w_{0,j}^1 = 0, and w_{0,j}^2 = 0.

Remark The proof is illustrated visually by Figure 9 (the weights equal to 0 have not been represented). On a side note, the HFFF can also model differences in EWMAs: simply slightly change Figure 9 into Figure 10. There are three different ways to come up with the exact results when handling Figure 10. We will address the problem of rigorously formalizing mathematically what constitutes a trend following in a subsequent article. However for now, in order to keep things intuitive, we will consider a TF strategy to have a topological DNA which would look like the one from Figure 9.

2.2.2 Multi-linear regression topology

Multi-linear regression (MLR) is another well-known 101-type strategy traders have been using in the industry.

Figure 9: The EWMA strategy translated in terms of network topology (the weights equal to 0 have not been represented)

![Image of a network diagram](image)

\[16\] Note that there are different ways to achieve the same numerical results though with a different topology.
2.2.3 XOR topology

We recall here the truth table associated with the XOR function in Table 1. How is this relevant to HFT? Let’s look at the following known HF rational.

Definition. If we define the open interest (OI) as being the total volume left on the order book then it is known that when:

- the price and the OI are rising, the market is bullish;
- the price is rising but the OI falling, the market is bearish;
- the price is falling but the OI rising, the market is bearish;
- the price and the OI are falling, the market is bullish.
Figure 13: The XOR strategy translated in terms of network topology

Table 1: The truth table of the XOR function

<table>
<thead>
<tr>
<th>I₁</th>
<th>I₂</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: The relationship between open interest, price, and Signal

<table>
<thead>
<tr>
<th>Price</th>
<th>Open Interest</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rising</td>
<td>Rising</td>
<td>Buy</td>
</tr>
<tr>
<td>Rising</td>
<td>Falling</td>
<td>Sell</td>
</tr>
<tr>
<td>Falling</td>
<td>Rising</td>
<td>Sell</td>
</tr>
<tr>
<td>Falling</td>
<td>Falling</td>
<td>Buy</td>
</tr>
</tbody>
</table>

Remark These four market situations can be summarized by Table 2.

Proposition The HFFF can model XOR-like strategies.

Proof. Simply set $\cup_{j \in [1,4]} w_{s_j}^{I_1} = 0$, $\cup_{j \in [1,4]} w_{s_j}^{I_2} = 0$, $\cup_{j \in [5,9]} w_{s_j}^{I_1} = 0$, $\cup_{j \in [5,9]} w_{s_j}^{I_2} = 0$, $w_{s_1}^h = 0$, $w_{s_3}^h = 0$, $w_{s_1}^l = 0$ and $w_{s_3}^l = 0$.

Remark We will make the following two observations:

- The preceding proof is visually illustrated by Figure 14 (the weights equal to 0 have not been represented).
- The XOR topology can be designed in various ways.\(^{17}\)

2.2.4 Execution strategy

To make the problem more realistic, one needs to formalize an execution strategy rule which would apply to all strategies but still be rule based and a function of its topology. In this first article we take the simple approach in which all strategies will follow Algorithm 1. The idea of this algorithm will be that:

- the execution strategy will be subject to a certainty-like function;
- certainty will be decided by the historical returns from the relevant topology split into intervals;
- since the decision needs to be made and that data comes regularly, a rolling percentile function should be used.

In this context algorithm PERCENTILE($D_{t-1}^I$, $\alpha_1$, $\alpha_2$) returns a value between 0 and 9, the nine pillar points for our order book. The tested input is compared against the $\alpha_1$ and $\alpha_2$ percentiles. Given that no history exists in the first iteration and that the first few iterations are not significant, we will randomize the first $R_n$ iterations (though this is not mentioned in Algorithm 1).

Algorithm 1 EXECUTION STRATEGY($T$, $I_{t}$, $D_{t}$)

Require: topology $T$, array of current inputs $I_{t}$, array of rough previous execution decisions $D_{t-1}^I$.

Ensure: strategy of topology $T$ modifies the order book by putting an order $O$ at any of the 9 positions of the order book.

1: \{Side comment: $A_i$ for asked price at i bps from mid\}
2: \{Side comment: $B_j$ for bid price at j bps from mid\}
3: $D_{t}^I \leftarrow$ CALCULATE($T$, $I_{t}$)
4: if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, 0, $\frac{1}{3}$) then
5: \hspace{2em} $O \leftarrow B_4$
6: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{1}{3}$, $\frac{2}{3}$) then
7: \hspace{2em} $O \leftarrow B_3$
8: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{2}{3}$, $\frac{5}{3}$) then
9: \hspace{2em} $O \leftarrow B_2$
10: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{5}{3}$, $\frac{8}{3}$) then
11: \hspace{2em} $O \leftarrow B_1$
12: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{8}{3}$, 9) then
13: \hspace{2em} $O \leftarrow A_1$
14: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{8}{3}$, $\frac{5}{3}$) then
15: \hspace{2em} $O \leftarrow A_2$
16: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{5}{3}$, $\frac{2}{3}$) then
17: \hspace{2em} $O \leftarrow A_3$
18: else if $D_{t}^I \in$ PERCENTILE($D_{t-1}^I$, $\frac{2}{3}$, 1) then
19: \hspace{2em} $O \leftarrow A_4$
20: else
21: \hspace{2em} $O \leftarrow m$ \{m for this is a comment\}
22: end if
23: return $O$

\(^{17}\)We will address the problem of rigorously formalizing mathematically what constitutes an XOR strategy in a subsequent article. However for now, in order to keep things intuitive, we will consider a XOR strategy to have a topological DNA which would look like the one from Figure 13 or 14.
3 Genetic algorithm as a mean to study the market through time

In this section we will specify the genetic algorithm which we have used to study our problem. Throughout this subsection we will refer to micro and macro increments.

Definition We define two types of iterations:

- the first type being micro corresponding to an infinitesimal increment in our environment, namely an increment in which a strategy $S$ analyzes and in turn changes the order book by placing an order itself;
- the second type being macro corresponding to a generational increment in our environment, namely a certain equal number of micro increments per strategy leading to a calculation of P&L and a survival process based on this P&L.

We will label by $N_k$ the number of total alive strategies, $N_k^\text{TF}$ the number of TF-like strategies, $N_k^\text{MLR}$ the number of MLR-like strategies, $N_k^\text{XOR}$ the number of XOR-like strategies, and $N_k^\text{others}$ the number of other unclassified strategies. The relationship between these entities can be summarized by equation (6):

$$N_k = N_k^\text{TF} + N_k^\text{MLR} + N_k^\text{XOR} + N_k^\text{others}$$

A strategy will consist of a topology $T$, a rolling P&L $P$, and a common order book $\mathcal{O}$ as shown by equation (7):

$$S \triangleq \{ P, T, \mathcal{O} \}$$

Remark One may ask why we have not chosen the first letters of each of the strategies (“t” for trend following, “m” for multi-linear regression, and “x” for XOR strategy). The reason they have been named this way is because, as we will see in Section 4.3:

- $N_k^\text{TF}$ behaves in mathematical biology like the number of preys in a Lotka–Volterra (LV) three-species equations [3];
- $N_k^\text{MLR}$ behaves in mathematical biology like the number of mixed (both prey and predator) in an LV three-species equations;
- $N_k^\text{XOR}$ behaves in mathematical biology like the number of super predators in an LV three species equations.

The different possible permutations, constraints on the first letters being different for each type of strategy, and the association with the LV three-species equation made the choice of $e, m,$ and $r$ at first glance the most optimal in this qualitative optimization by constraint problem.

3.1 Survival and birth processes

The survival, death, and birth processes are a set of processes which impact the number of live strategies $N_k$ at any time $k$ the following way. If we call $S_{(n)} = S_{(n)}^1, S_{(n)}^2, \ldots, S_{(n)}^n$, the strategies ranked with respect to their P&L from highest to lowest, we will admit the following definitions:

Definition The Survivor set is the set of strategies with a positive P&L. Namely if $S_k = S_{(1)}, S_{(2)}, \ldots, S_{(n)}$, with $S_{(1)} \geq 0$ and $S_{(n+1)} < 0$. We will subdivide this set by distinguishing:

- secondary survivors set with carnality $a_2 = \lfloor \frac{s}{2} \rfloor$, survive without reproducing;
- primary survivors set with carnality $a_1 = s - a_2$, survive and have one offspring with a “slightly different DNA” in the form of a conditional resampling of their topology.

Definition We will call the Birth process, the first half of survived strategies. Namely, if $a_1 = b = \lfloor \frac{s}{2} \rfloor$, the strategies $S_1, \ldots, S_n$ will both survive and reproduce and create a set of equal size but with a slightly different topology and with carnality $b = a_1$.

Definition We will call the Death process, the set of strategies with a negative P&L. Namely if $S_k = S_{(n+1)}, S_{(n+2)}, \ldots, S_{(n+j)}$, MLR-disappear from the market at macro iteration $k + 1$.

Remark We can easily see that $s = a_1 + a_2, a_1 \geq a_2, a_1 = b$. Figure 15 illustrates these few definitions.

3.2 Inheritance with mutations

The intuition about the mutation process is that each birth is a function of a successful strategy (the positive P&L of parents $S_1, \ldots, S_n$) and should resemble a great deal that single parent which produced it but be at the same time a bit different to allow the ecosystem to evolve. We have seen in Section 2 that the DNA of our strategies is essentially their topology $T$ (which is itself a combination of weights). We will therefore concentrate on performing resampling on the weights of the offspring. Recall that the pdf of the beta distribution is given by

$$Beta(x; a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1 - x)^{b-1}$$

18Explained next.
19This label will be the same in Section 4.3.
20Or alternatively alive process.
21So no crossover in this model.
Figure 15: Illustration for the Death and Birth processes in our GA

Figure 16: PDF of the beta distribution for different combinations of \((\alpha, \beta)\)

with \(\Gamma(n) = (n - 1)!, 0 \leq x \leq 1\), and \(\alpha, \beta > 0\), the shape parameters. The reason why this distribution is interesting is that it is defined in a closed interval \([0,1]\) and can therefore be rescaled easily through a change of variable to \([-1,1]\), an interval which is a basic way of formalizing a normalized importance of each node in the topology decision making of Figure 8. It also offers a broad range of interesting shapes, allowing the possibility to code a conditional resampling model and therefore make clever proximity changes around the symbolic levels: \(-1, 0,\) and \(1\). We can see how the shape parameters can achieve these targeted resamplings in Figure 16. This way we can prevent too large deviations and rather select small incremental changes and intuitively follow the principles of selection. We can see that \(Beta(x, 1, 7)\) and \(Beta(1-x, 1, 7)\) both concentrate a great deal of the distribution toward 0 and 1, respectively. Likewise \(Beta(x, 3, 7)\) and \(Beta(x, 5, 7)\) provide a more Gaussian-like distribution toward in-between zones, which is what we want:

\[
D(\bar{x}) = \mathbf{1}_{\bar{x} \leq 1} Beta\left(\frac{\bar{x} + 1}{2}; \alpha(\bar{x}), \beta\right)
+ \mathbf{1}_{\bar{x} > 1} Beta\left(1 - \frac{\bar{x} - 1}{2}; \alpha(\bar{x}), \beta\right)
+ \mathbf{1}_{|\bar{x}| \leq 1} f(\bar{x})
\]

\[
\alpha(\bar{x}) = \begin{cases} 1, & \text{if } 1 > |\bar{x}| \geq \frac{1}{4} \\ 3, & \text{if } \frac{3}{4} > |\bar{x}| \geq \frac{1}{2} \\ 5, & \text{if } \frac{1}{2} > |\bar{x}| \geq \frac{1}{4} \end{cases}
\]

\[
F(k) = \begin{cases} 0, & \text{if } k \leq -\frac{1}{4} \\ \frac{k}{4}, & \text{if } |k| < \frac{1}{4} \\ 1, & \text{if } k \geq \frac{1}{4} \end{cases}
\]

with \(\bar{x} \in [-1, 1], \beta = 7\), and the function \(\alpha(\bar{x})\) modeling the interval of condition, arbitrarily chosen, though constructed by noticing that the mode of the beta distribution is given by \(\frac{\alpha - 1}{\alpha + \beta - 2}\) and also so as to make the fractions easy and the intervals loosely equal.

4 Results, parallel to predator–prey models, and evolutionary dynamics

4.1 Observations and interpretation

4.1.1 Results

Many improvements can be made with the coding exercise since it proved to be a challenging task, however, with some of the idiosyncratic simplifications used on
4.1.3 Interpretation

We propose the following interpretation for the observations from Figure 17:

- **TF strategies** are what people commonly call self-fulfilling prophecy-like strategies, meaning that they only work as long as everyone making up the competitive environment follows the same trend. The biological mirror as described from Section 4.3 would be an ultimate prey which, given an environment without any predator, would never die and actually grow exponentially.
- **The XOR strategy** is a super predator strategy (similar to the $z$ parameter in Section 4.3) and feeds on the MLR strategies.
- **MLR are both predator and prey strategies**, feeding on the TF strategies but used as prey by the XOR strategies.
- **The way the MLR dominates the TF strategy** is due to the fact that it looks at additional leading information on the order book (the volumes at the different depths of the order book) so it is leading the trend, whereas the TF is lagging the trend.
- **XOR strategies can only survive if enough preys (MLRs) are present in the ecosystem**, otherwise it dies.
- The way the XOR strategy dominates the MLR strategy is due to its ability to hide its cards better and be able to better decipher spurious positions at higher depths of the order book.
- **The XOR strategy cannot invade the TF strategies on its own** since the sophistication of its bait (the systematic strategy built to bait the MLR) is too complex to trick the TF. An analogy could be made with a professional poker player playing with a beginner whose moves are almost random.

4.2 Comparison to game theory

In this section we present a few results from the world of game theory in order to make the parallel to our results for a better understanding of how these strategies interact with each other.

4.2.1 Prisoner’s dilemma

The prisoner’s dilemma (PD) is a well-known game theory introductory concept. The way it is usually explained is that a couple of criminal associates are taken into separate rooms and independently interrogated. The prosecutor wants to close the case and send someone to jail, so he offers a deal to both captives. If the criminals both cooperate (C), nobody goes to prison but they each get a heavy fine. If one denounces/deceits (D) the other, then he will be free without any fine, but the one being denounced has to go to prison and get a fine. If they each denounce each other, they go to prison without a fine. Broadly speaking that little story can be formalized into a $2 \times 2$ matrix of $CC, CD, DC,$ and $DD$ with respective payoffs $(2,2), (0,3), (3,0),$ and $(1,1).$ The reason why this game theory concept is within the family of dilemmas is because although the prisoners clearly should cooperate here, given that they do not know what the other is going to do, by expectation (with equal probability for a C and a D) any user should deceit given that the expectation of the payoff for a deceit is 2 as opposed to 1 for a cooperation.

4.2.2 Axelrod’s computer tournament

The dilemma presented in the previous subsection proved to shuffle the rules of payoff strategy optimality when the game became iterative. Robert Axelrod was a main contributor to this field. Indeed, Axelrod [1, 2] designed a computer
tournament aimed at looking at what strategy would prevail in an iterative format. On that occasion he invited a few mathematicians, computer scientists, economists, and political scientists to code a strategy they believed could win such a tournament with the constraints of PD rules in which it is not known when the tournament will stop. Many strategies were thrown into this ecosystem in the form of a tournament ranging from being simplistic like “Always Deceit” (AD) strategy to many other more complicated strategies; a generic representation is shown in Figure 18(b). Surprisingly the Tit For Tat (TFT) strategy came at the top of this tournament. The TFT is considered in the literature to be a nice strategy, meaning that it is never the first to deceit (its first move is by design to be a C), but it is also a strategy that is able to retaliate in a situation in which it was previously deceived. Finally, it is a strategy that is able to forgive, meaning that if it sees that the adversary has decided to cooperate after a deceit, then it switches back to a C.

4.2.3 Evolutionary dynamics

Martin Nowak [21] recently enhanced some of Axelrod’s work by introducing new strategies and further developing the concepts of invasion/dominance within a competitive strategic ecosystem. For instance, as we can see from Figure 18(d), some strategies invade others but these latter strategies can be in turn invaded by other ones, which in turn can be invaded by the very first strategy mentioned and induce cycles. Indeed an ecosystem composed of a set of unbiased random strategies (that would randomly select C or D) would invite the invasion of an ALLD (always defect) kind. In turn the frequency of ALLD would take the ecosystem, which would invite the TFT strategy, which would benefit from the mutual cooperation when within the same proximity, etc. Figure 18(d) exposes how some of these strategies may interact with each other. The following additional information may help in refreshing what some of these acronyms mean:

- TFT (Tit For Tat) developed in the previous section
- GTFT (Generous Tit For Tat) which makes it slightly less grudge-prone compared to the TFT as it only deceits for two successive Ds from the opponent
- WSLS (Win-Stay, Lose-Shift) that outperforms TFT in the prisoner’s dilemma game [21, 28]
- ALLD (Always Deceits) which is self-explanatory
- ALLC (Always Cooperates) which is also self-explanatory
- rand (Random Strategy) which outputs a C or a D with equal probability.

The main takeaway from this parallel was to expose how the rise and fall of strategies can easily be engineered through simple systematic rules based on an ecosystem and how complexity can be induced from simple rules.

4.3 Theoretical biology and predator–prey models

It was discussed in the 1960s [9] that complexity in an ecosystem insures its stability or to keep the same jargon “communities not being sufficiently complex to damp out oscillations” [7, 11] have a higher likelihood of vanishing. It is widely accepted, in the context of ecosystem simulation, that complexity should always arise from simplicity [4, 17].

4.3.1 Literature review

The diversity–stability debate in the context of ecosystem modeling has been ongoing since the 1950s [18] with no consensus ever being reached. It was initially believed [6, 14, 18] that given that nature was infinitely complex a more diverse ecosystem should insure more stability. This assertion was however ultimately challenged through rigorous mathematical specification [17, 22, 32] in the 1970s and 1980s by using LV’s predator–prey model initially published in the 1920s [13, 31] with similar “non-intuitive” results. More recently the work has been extended to small ecosystems of a three-species food chain [3]. The intuitive three-species example we have chosen to discuss is the one containing sharks (chosen to be the x parameter), tuna (chosen to be the y parameter), and small fishes (chosen to be the z parameter), the idea being that tunas eat small fishes which in turn are eaten by sharks. Without loss of generality sharks are assumed to die of natural causes and their decomposing bodies go on to feed the small fishes.

The set of differential equations has been summarized in equation (12):

\[
\begin{align*}
\frac{dx}{dt} &= ax - bxy \\
\frac{dy}{dt} &= -cy + dxy - eyz \\
\frac{dz}{dt} &= -fx + gyz
\end{align*}
\]
where \( a \) is the natural growth rate of species \( x \) in the absence of predator, \( d \) the one of \( y \) in the absence of \( z \). We also have \( b \) representing the negative predation effect of \( y \) on \( a \) and \( e \) the one of \( z \) on \( y \). We also have \( g \) which mirrors the efficiency of reproduction of \( z \) in the presence of prey \( y \). Note that we assume that \( x \) never dies of natural causes (if it’s too old then it can’t run fast enough to outrun predator \( y \)) but this is not the case for \( z \), since it is an alpha predator and therefore needs some natural death rate which is symbolized by \( f \). This relatively simple system of three equations has been studied extensively [18] for stability. For example, Figure 22 later represents a particular instance in which the system is unstable. Indeed, we can notice that the oscillations between the three species increase to the point, not shown here, where the amplitudes are so big that \( z \) goes extinct and at which point \( x \) and \( y \) start oscillating, with however a constant amplitude. We refer the motivated reader back to the original paper [18] for the other cases and interesting idiosyncratic properties. One interesting point to notice is that in cases of “relative best stability”, in which \( a = b = c = d = e = f = g = 1 \) % from Figure 19, we have oscillations which are stable through time with the highest peak from the ultimate prey \((x)\) coming first with the highest peak and then the one of the ultimate predator \((z)\) coming last but with the smallest amplitude. This suggests that sophisticated working trading strategies\(^{27}\) need enough prey-like strategies\(^{28}\) in the same ecosystem to get them to be profitable. One other interesting observation is that the total ecosystem population as depicted by the black line in the same figure suggests that it itself oscillates, which may not necessarily be intuitive. Indeed one could have speculated that the loss of a species directly benefits the others and that therefore the total population should stay constant. This interesting observation suggests that the oscillations of a financial market may likewise be subject to similar dynamics: a financial ecosystem may go through periods in which it thrives followed by periods in which it declines, the economy itself is cyclical with, some may argue, oscillations which are more and more important like the one depicted by the unstable ecosystem in Figure 22. The stunning similarities of the competitive resource-driven biological ecosystem along with some compelling similarities in some of its cyclical behavior makes the LV \( n \)-species food chain equation an interesting candidate when it comes to studying the stability of the financial markets especially the electronic trading markets, because of its systematic rule-based approach and zero-sum game-like roots.

**Remark** Note that these results corroborate some of the connections between utility functions and the LV model discussed recently [24, 26, 27].

### 4.3.2 The interesting case of the simplified four-species stochastic Lotka–Volterra

If the reader is not entirely convinced of the results from Figure 17, more specifically how these results can lead to the kind of real observations from Figure 1 since the full formalization and boundaries of the three strategies are not provided and therefore the analysis of Section 4.1.3 is put in question, let’s look at the following simplified stochastic system which is essentially a transform of the four-species LV. The transforms are listed below:

1. Assuming that the P&L lost by a strategy is linearly gained by another leads us to assume that \( a = a_i = \ldots = a_i = 1 \) and \( b = b_i = \ldots = b_i = 1 \).
2. The regulators may assume that the maximum number of strategies \( M \) can be fixed to the number of market participants: here we have chosen four.\(^{29}\) This is obviously arguable on the basis that a market participant may have multiple strategies but we can assume that this latter multiple strategy is itself a strategy.
3. Also we may assume that the notional associated with a certain family type may impact the market. For instance you may assume that a new market participant may enter and impact the market by entering it at a particular level of predation. This means that the types of strategies the participant may choose to implement may be in our case:

   - XOR type of strategy (with positive jump in notional upon entry \( \varphi_x \))
   - MLR type of strategy (with positive jump in notional upon entry \( \varphi_y \))
   - TF type of strategy (with positive jump in notional upon entry \( \varphi_y \))
   - unclassified type of strategy (with positive jump in notional upon entry \( \varphi_x \))

with \( \varphi_x \sim \chi_{e<0.01} \epsilon \) with \( \epsilon \sim U[0, 1] \). These simplifications give equation (13):

\[
\begin{align*}
\frac{dx}{dt} &= x - xy_1 + \varphi_x \\
\frac{dy}{dt} &= xy_1 - y_1y_2 + \varphi_y \\
\frac{dz}{dt} &= y_1y_2 - y_2y_3 + \varphi_y \\
\frac{dt}{dt} &= -z + \varphi_x
\end{align*}
\]

Figure 20 is a simulation from this “mirror simplified HFTE model”. We can see notice from 6000 until zone 10,000 a very similar situation with regular oscillations followed by a crash. If we increase the timescale (Figure 21) we actually see that periods of crashes are triggered by an \( \varphi_i > 0 \) with \( i \in \{x, y_1, y_2\} \), followed by an increase in frequency from the ultimate predator strategy \( z \) before the correction in the market occurs.

\(^{27}\) Perhaps from top algorithmic desks in top-tier investment banks?

\(^{28}\) Perhaps the retail clients of the world?

\(^{29}\) In the \( n \)-species simplified stochastic Lotka–Volterra, the \( n = 2 \) and \( n = 3 \) behave a bit differently but the case \( n \geq 4 \) is a family on its own when you take a look at the "market" level.
5 Application and summary

5.1 Application

Many applications could be implemented and we may disclose a few more in a subsequent article but for the sake of keeping this article relatively short we propose a couple of applications. The first of these applications will be in trading, which methodology will be expounded in Section 5.1.1; the second application we hope to be a new tentative approach in doing risk management at the regulatory level in Section 5.1.2.

5.1.1 Trading application

In terms of providing market context, the arrival of HFT on the Chinese commodities market has opened up a great number of opportunities at the systematic trading end because:

- Commodities have historically been a physical trading asset class and the arrival of HFT in commodities is rather new compared to equities and FX, therefore opportunities for making money are still quite vast.
- Regulations on the Chinese market make the life of a high-frequency trader much easier than in heavily regulated regions like Europe or the Americas.

In terms of providing additional information on the data: the data used consists of a week of data split into 12 months for 14 different commodities futures provided by a reputable HFT firm. In order to eliminate the survival bias, 15 random time series were chosen in the month of December 2015 and 15 other random ones were chosen in the month of November 2015. All the major commodities were represented in these 15 random time series, however few commodities seem to have limited data. Given that there is a bit of learning associated with any machine-learning exercise of this type, the commodities set that had little data was
eliminated. In terms of limitations: the data seems to be generally speaking of good quality however a few limitations were observed.

- First the data comes in every 500 ms, which means some important information may be lost.
- Some time series have too few data on one day to do proper training (those have been dismissed in the backtesting for now).
- Some files seem to have corrupt data though more investigation is necessary in that domain (for now those red-flag files have been dismissed).

If we were to provide the current limits, we would say that the trading signal used is very unsophisticated and can easily be improved by:

- better formulating the trend and trend reversal signals
- better modeling the co-movement of the different futures with respect to each other
- better optimizing the signal with respect to the cost process.

However, despite these limitations, as we can see from Table 3, the in and out-of-sample statistical performances are highly unlikely to have come about by chance.

5.1.2 Regulatory implications
The second and last immediate application we will take a look at in the context of this article is the one of systemic risk. Given that this article proposes that the fluctuations of the markets are linked to the frequency of the strategies composing the ecosystem of the market, we propose a model which would take advantage of this assumption to build new approaches in doing high-level regulations. The exercise would consist of monitoring these strategies’ interactions and flagging this as an indication of the need to build new approaches in doing high-level regulations.

Remark The work around guessing could actually be as simple as asking the market participants to provide the code of their systematic trading strategies under the motivation of national interest and for the sake of the stability of the financial markets. In exchange, the regulators would agree on keeping the strategies confidential. This may raise ethics questions as the concept of risk would dangerously flirt with the concept of “strategy destiny” as the regulators would be able to analyze which market participant would end up profiting and losing from the environment before these losses actually occurred.

Now going back to the actual mathematical study of the stability of the financial market. Answering if the financial market is stable would come to studying the Jacobian matrix \( J \) from equation (14):

\[
f(x, y, z) = \begin{bmatrix}
    a - by & -xb & 0 \\
    yd & -c + dx - ez & -ye \\
    0 & -g & f + gy
\end{bmatrix}
\]

By examining the eigenvalues of \( f(x, y, z) \) we can indirectly gain information around the equilibrium of our financial system at the regulatory level. More specifically, if all eigenvalues of \( f(x, y, z) \) have negative real parts then our system is asymptotically stable. Figure 22 gives an illustration of a situation in which one of the eigenvalues is negative. Many questions could be raised here: how can the regulators gain information on the parameters composing the systems of equation (12)? Also the market has surely more than three types of strategies, how many exactly? Are these strategies easily classifiable in terms of prey, predator, and super predator? It is very likely that trading desks, especially in the high-frequency domain, refuse to provide their sets of strategies for the regulators to study. The Jacobian matrix methodology cannot therefore be studied. Bespoke relevant actions may no longer be taken. This is where the Section 6.2 on guidance for future research has been added. In the meantime though, in order to encourage the motivated reader to think about the problem of stability in the financial markets, we introduce the following conjecture:

Conjecture Diversity in financial strategies in the market leads to its instability.

Remark Note that this conjecture can be studied indirectly or at least intuitively using some of the findings in mathematical biology. More specifically the one associated with diversity in ecosystems and stability.

6 Summary

6.1 Summary
We started this article by pointing to a puzzling observation from the newly born high-frequency commodities market which, because of its extreme youth and therefore immaturity, makes it a great case study for a high-frequency market at inception and therefore for our purpose. More specifically, as we have seen from Figure 1, on June 8, 2011, a fascinating patterned oscillation occurred on the commodities market. We have proposed in this article that these oscillations are due
to the interactions of the different strategies participating in the market and contributing to the fluctuations of the market. We have proposed that these oscillations are of the same nature as the LV model. In order to test our hypothesis we have proposed a topology which we proved is able to model the HFT strategies known to the market participants. More specifically, we have illustrated how it can achieve the TF, TM, and XOR strategies. This topology is then randomly sampled at inception (the random seed) in this swarm market and a simple genetic algorithm is enforced to allow us to study the market and its participants through time. The results are commensurate with the LV model as well as some of the other game theory results, more specifically around strategy invasion that we have also made a comparison to. Finally, we have applied our findings to a simple trading exercise as well as proposed a new way to monitor the markets for regulatory purposes. Table 4 summarizes roughly these findings.

### 6.2 Guidance for future research

#### 6.2.1 The regulatory aspect

As we have seen in Section 5.1.2, the regulatory implications from this research naturally invite us to explore a research project in which we would try to guess\(^{33}\) the frequency of each type of strategy using the LV multi-species models as likelihood functions. We propose to use a particle filter on scenarios to achieve this point. We will discuss this particular point in a subsequent article.

#### 6.2.2 The options market

We have recently introduced a new parametrization of the implied volatility surface\(^{[5, 16]}\) and have established that de-arbing is a convoluted mathematical optimization whose simplification can be illustrated in Figures 23 and 24. For the sake of making the notation a bit more intuitive, we use the notation from Table 5, where equation (15) provides the relevant equivalences:

\[
\begin{align*}
P_{t-1} &:= C_i(K e^{-\gamma \Delta}, T - \Delta) \\
P_{t-1} &:= C_i(K e^{-\gamma \Delta}, T) \\
P_{t} &:= C_i(K - \Delta e^{-\gamma \Delta}, T) \\
P_{t+1} &:= C_i(K + \Delta e^{-\gamma \Delta}, T) \\
P_{t+1} &:= C_i(K, T + \Delta)
\end{align*}
\]

Table 4: Rough summary of the different model discussed

<table>
<thead>
<tr>
<th>Ecosystem Size</th>
<th>Main Players</th>
<th>Main Results</th>
<th>Main Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Strategies</td>
<td>&quot;sort of&quot; TF, MLR, and XOR</td>
<td>parallel to LV three-species predator–prey model</td>
<td>Maps between network topology and strategy families need to be better specified</td>
</tr>
<tr>
<td>4 Strategies</td>
<td>Above + random strategy</td>
<td>Stability of the market can be studied using the parallel mathematical biology problem</td>
<td>Same as above</td>
</tr>
<tr>
<td>n Strategies</td>
<td>Above + sets of Ignoratus, Praedor, and Servus Dominum</td>
<td>More complex LV with food web structure + potential strategy with foresight and farming</td>
<td>Diversity in financial strategies in the market lead to its instability?</td>
</tr>
</tbody>
</table>

Here \(C_i(\cdot)\) represents the call price under the relevant asset class assumption.\(^{34}\) We aim to study the Bayesian probability problem of equation (16):

\[
P \left( \mathbf{P}^l \mid l(\mathbf{P}^l) \right) 
\]

where \(F = \{P_{i}^{l_1}, P_{i}^{l_2}, P_{i}^{l_3}, P_{i}^{l_4}, P_{i}^{l_5}\} \) in the discrete space, \(P = P_{i}^{l_1}\) and \(l\) represents the lag-inducing function such that \(l(P_{i+1}) = (P_i)\). The implied volatility is a very different product than spot because it has a tendency to mean revert, it is very much subject to what the adjacent points are doing and reacts in a lower frequency than spot. Our aim will be to study the HFTE in light of these observations. However, it is interesting to already notice that these observations could be addressed by a modification of the HFFF (Figure 25). Following the rationale of Section 2, we need to create a learning architecture that would incorporate the following observations:

33 Given that we cannot ask the market participants to provide us with their strategies.

34 For example, log-normal diffusion in equities, normal diffusion for rates, and Garman-Kohlhagen for FX.

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Figure 24: Visualization for the core simple de-arbing idea

- Presumably the price point \( p_t^{ij} \) can be best approximated by the four adjacent points, a simple MLR can be used to model this idea (green part of Figure 25).
- The second point to notice is that each point of the implied volatility is a mean-reverting stochastic process and this can be modeled in terms of network architecture by a spread of EWMA (blue part of Figure 25).
- At least one hidden layer to address some of the economical drivers, leading to a need for an architecture that could learn XOR-like functions like we saw, could sometimes be necessary in algorithmic trading from Table 2 (red part of Figure 25).

Remark: Note that the XOR-like functions may not be as necessary as the dynamics of spot since vol may be driven by economical factors that are different, especially if we study the problem at different timescales. This suggests that the red part of Figure 25 may, in the end, be the identity function. For the sake of keeping that door open though, we have left it in our network topology.

We will also see how the parameters introduced in the newly published IVP model [16] can contribute in fine tuning the learning process as well as its execution strategy, which requires insight around liquidity.

6.2.3 Network topology for implied volatility point dynamics learning
Following the rationale of Section 2, we need to create a learning architecture that would incorporate:

6.3 Possible strategy classification
As we have seen in Section 2, the formalization of these pure prey, mixed prey/predator, and pure predator strategies needs to be more rigorous.


Table 5: Naming grid associated to figure 24

<table>
<thead>
<tr>
<th>( p_{t-1}^{ij} )</th>
<th>( p_t^{ij} )</th>
<th>( p_t^{ij+1} )</th>
</tr>
</thead>
</table>

6.3.1 Guidance on strategy naming
The tentative naming inspiration initially came from:

- exploring how ecologists have, by convention, named the different species in Latin (Homo Habilis, Homo Neanderthalensis, Homo Sapiens, etc)
- noticing that the naming was descriptive (e.g., Homo Habilis is supposed to have used tools)
- noticing that being extinct does not necessarily mean a species would not have been dominant today in certain conditions (for instance it’s not hard to imagine that, had the Homo Sapiens never been born, then Homo Neanderthalensis would probably have been able to enslave other animals such as cattle in almost the same way we do. Therefore an ecological niche supersedes any human conception that only the most “superior” species prevail.

6.3.2 Core naming conventions
It seems that strategy in Latin gives Militarium and that ancestor is Antecessoris, therefore:

- Ignoratus, for example the Militarium Ignoratus would essentially correspond to a random strategy without much “intelligence” (for example a TF in finance) corresponding, in a biological ecosystem, to perhaps a vegetarian (e.g., mouse) in an advanced ecological world.
- Praedor, for example the Militarium Praedor would correspond to a strategy that can dominate the Militarium Ignoratus-like species. Note that a few Militarium Praedor could be in competition at one point, which could ultimately lead to one of the Militarium Praedor going extinct. The analogy to our world could be one in which leopards and lions both compete for the antelope in Africa but lions are slightly better at it and which may lead ecologists to speculate that the leopard is now an endangered species. One
type of well-known extinct ancestor of the big cats is the saber-toothed tiger. The Militarium Praedor Antecessoris in our ecosystem would correspond to that leopard who would have gone extinct.

- **Servus Dominium**, for example the Militarium Servus Dominum would correspond to a strategy that can dominate and have foresight with respect to the Militarium Ignoratus or/and the Militarium Praedor-like species. An example of such a biological ecosystem could be one in which humans\(^1\) would for example feed a fox\(^2\) population with mice\(^3\) in the context of fur farming. Can a strategy have so much foresight and understanding of the market that it can implement this idea on the markets? We will address these questions in subsequent articles.

Babak Mahdavi-Damghani has been working in the financial industry within a broad range of functions (Exotics and High-Frequency Trading, Structuring, FO and Risk Quantitative Analytics, Clearing), through all major asset classes (Equities, Commodities, FX, Rates, Hybrid) in both the buy and sell sides across different geographical locations. His undergraduate education took place at the University of Pennsylvania in what would now correspond to Financial Engineering. His post-graduate studies were accomplished in several areas of Applied Mathematics and Computational Sciences at the University of Oxford. He is the founder of EQRC and currently doing research at the Oxford Man Institute of Quantitative Finance in the area of Machine Learning. He is also the author of numerous publications, including cover stories of Wilmott magazine and mathematical models (e.g., Cointelation and NVP) that are now taught in CQF. His current research is in the application of Machine Learning to Quantitative Finance, broadly subdivided into three themes going from electronic trading to dimensionality handling in the context of options modeling and quantitative risk modeling.

**References**


\(^3\)By analogy, *Militarium Servus Dominum*.
\(^4\)By analogy, *Militarium Praedor*.
\(^5\)By analogy, *Militarium Ignoratus*. 
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