Market Risk Premium in Power Markets

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1. Explaining the Spot-Forward Relationships
   - Classical Theory
   - Market Risk Premium

2. Information Approach
   - The Information Premium
   - The information premium with delivery period

3. Empirical Study
Agenda

1. Explaining the Spot-Forward Relationships
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Spot-Forward Relationship: Classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

\[ F(t, T) = S(t)e^{(r-y)(T-t)} \]  

(1)

where \( r \) is the interest rate at time \( t \) for maturity \( T \) and \( y \) is the convenience yield.
Spot-Forward Relationship: Classical theory

- In a stochastic model this means

\[ F(t, T) = \mathbb{E}_Q(S(T)|\mathcal{F}_t) \]

where \( \mathcal{F}_t \) is the accumulated available market information (in most models the information generated by the spot price).
Spot-Forward Relationship: Classical theory

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where \( \mathcal{F}_t \) is the accumulated available market information (in most models the information generated by the spot price).

- \( Q \) is a risk-neutral probability
  - discounted spot price is a \( Q \)-martingale
  - fixed by calibration to market prices or a market price of risk argument
Forward selling¹ by RWE Power in the German market

(Base-load forwards in €/MWh)

(average realised price for 2008 forward: €58/MWh, for 2009 forward: €70/MWh)

¹ Forward prices until November 8, 2010; hedge ratio as of Sept. 30, 2010.
Spot-Forward Relationship

- Normal backwardation: Forward prices are below spot price – Producers accept paying a premium for securing future production because of hedging pressure for long term investments.
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- In electricity markets one normally observes that,
  - for ‘long’ dated forward contracts, markets are in backwardation (forward below spot)
  - for ‘shorter’ maturities the markets are in contango (forward above spot).
The market risk premium or forward bias $\pi(t, T)$ relates forward and expected spot prices.
Market Risk Premium

- The *market risk premium* or *forward bias* $\pi(t, T)$ relates forward and expected spot prices.
- It is defined as the difference, calculated at time $t$, between the forward $F(t, T)$ at time $t$ with delivery at $T$ and expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}^{P}[S(T)|\mathcal{F}_t].$$  

(2)

Here $\mathbb{E}^{P}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time $t$ and $S(T)$ is the spot price at time $T$. 
Equilibrium approach Benth, F.E.; Cartea, A.; Kiesel, R. *JBF*: 
Consider producers and retailers and their long- and short-term hedging pressures.
Market Risk Premium – Explanations

- Determine bounds for forward price by producer and retailer indifference prices.
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- Actual forward price is found by using market power.
Market Risk Premium – Explanations

- Determine bounds for forward price by producer and retailer indifference prices.
- Actual forward price is found by using market power.
- The model is able to reproduce a changing sign of the market risk premium.
Literature – Bessembinder/Lemmon


- Use an equilibrium model to analyse predictive power of forward price
Literature – Bessembinder/Lemmon


- Use an equilibrium model to analyse predictive power of forward price
- Find a downward bias if expected power demand is low and demand risk is moderate
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- Use an equilibrium model to analyse predictive power of forward price
- Find a downward bias if expected power demand is low and demand risk is moderate
- Forward premium increases if either expected demand or demand variance is high
- Some empirical evidence

- Empirical analysis of a high-frequency data set of day-ahead forward prices

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- Find a significant risk premium, which varies throughout the day and is related to volatility of demand and risk of price spikes
Literature – Diko et al.


- Empirical analysis of German, Dutch and French Energy Markets

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- Infer a term-structure of the risk premium
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- Empirical analysis of German, Dutch and French Energy Markets
- Infer a term-structure of the risk premium
- Identify skewness of spot prices and variability of spot prices as main factors driving the risk premium
Literature – Benth/Meyer-Brandis

Benth, F. and Meyer-Brandis, T.: The Information Premium in Electricity Markets; *Journal Energy Markets*

- Discuss assumption that information filtration is generated by spot prices
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- Use 'enlargement of filtration' to incorporate future information on the spot (such as power plant maintenance)
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- Discuss assumption that information filtration is generated by spot prices
- Use 'enlargement of filtration' to incorporate future information on the spot (such as power plant maintenance)
- Theoretical argument that a significant part of the market price of risk is due to an information premium
Agenda

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Information Approach

- As electricity is non-storable future predictions about the market will not affect the current spot price, but will affect forward prices.
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- Market example: in 2007 the market knew that in 2008 CO₂ emission costs will be introduced; this had a clearly observable effect on the forward prices!
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- Market example: in 2007 the market knew that in 2008 CO₂ emission costs will be introduced; this had a clearly observable effect on the forward prices!
- German moratorium 2011: shut-down 7 nuclear power plants for 3 months with possible complete shut-down.
German Moratorium I

Figure: EEX spot prices

Explaining the Spot-Forward Relationships Information Approach Empirical Study
The Information Premium The information premium with delivery period
German Moratorium II

Figure: EEX forward prices delivery May 2011
Figure: EEX forward prices delivery August 2011
Example: 2008 CO$_2$ Emission Costs

![Graph showing EEX Forward prices observed on 01/10/06 (left) and 01/10/07 (right).]

**Figure:** EEX Forward prices observed on 01/10/06 (left) and 01/10/07 (right)

- Typical winter and bank holidays behaviour in both graphs
- General upward shift in 2008

$\Rightarrow$ 2nd phase of CO$_2$ certificates
Future information is incorporated in the forward price
Information Approach

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- ... but not necessarily in the spot price due to non-storability
Information Approach

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- ... but not necessarily in the spot price due to **non-storability**
- ... buy-and-hold strategy does not work
Information Approach

- The usual pricing relation between spot and forward:

\[ F(t, T) = \mathbb{E}^Q[S_T | \mathcal{F}_t] \]

- Not sufficient: natural filtration \( \mathcal{F}_t = \sigma(S_s, s \leq t) \)
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- Idea: enlarge the filtration!
The usual pricing relation between spot and forward:

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Idea: enlarge the filtration!

... by information about the spot at some future time \( T_\gamma \)

Info could be that spot will be in certain interval...

... or the value of a correlated process (temperature)
Information Approach

Filtrations

- $\mathcal{F}_t$ - the historical filtration
- $\mathcal{H}_t$ - complete information, i.e. $\mathcal{H}_t = \mathcal{F}_t \vee \sigma(S(T_\gamma))$
- $\mathcal{G}_t$ - the filtration of all information publicly available to the market
Information Approach

Filtrations

- $\mathcal{F}_t$ - the historical filtration
- $\mathcal{H}_t$ - complete information, i.e. $\mathcal{H}_t = \mathcal{F}_t \lor \sigma(S(T_\tau))$
- $\mathcal{G}_t$ - the filtration of all information publicly available to the market

Hence, we have the relation $\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{H}_t$

In the following we will consider the observed forward as

$$F(t, T) = \mathbb{E}^Q[S(T)|\mathcal{G}_t]$$
The Information Premium

- Quantify the influence of future information using:

**Information Premium**

The information premium is defined to be

\[ I(t, T) = \mathbb{E}[S_T | G_t] - \mathbb{E}[S_T | F_t] \]

i.e. the difference between the prices of the forward under \( G \) and \( F \).
The Information Premium

Lemma

There is no information on the information premium in \( \mathcal{F} \).

Proof:

\[
\mathbb{E}[I_G(t, T) \mid \mathcal{F}_t] = \mathbb{E}[\mathbb{E}[S(T) \mid G_t] - \mathbb{E}[S(T) \mid \mathcal{F}_t] \mid \mathcal{F}_t] = 0
\]
The Information Premium

Lemma

There is no information on the information premium in $\mathcal{F}$.

Proof:

$$\mathbb{E}[l_G(t, T) \mid \mathcal{F}_t] = \mathbb{E}[\mathbb{E}[S(T) \mid \mathcal{G}_t] - \mathbb{E}[S(T) \mid \mathcal{F}_t] \mid \mathcal{F}_t] = 0$$

- Result valid for all measures equivalent to $\mathbb{P}$
- Usual method to attain the market price of risk is a measure change
- This is not possible for the Information Premium
Spot price model and forward price with delivery

Two-factor arithmetic Spot Price

\[ S(t) = \Lambda(t) + X(t) + Y(t) \]
\[ X(T) = e^{-\alpha(T-t)}X(t) + \sigma \int_t^T e^{\alpha(T-s)} dW(s) \]
\[ Y(T) = e^{-\beta(T-t)}Y(t) + \int_t^T e^{\beta(T-s)} dL(s) \]

where \( \Lambda(t) \) is deterministic, \( W(t) \) a BM, \( L(t) \) a Lévy process.
Spot price model and forward price with delivery

Two-factor arithmetic Spot Price

\[ S(t) = \Lambda(t) + X(t) + Y(t) \]

\[ X(T) = e^{-\alpha(T-t)}X(t) + \sigma \int_t^T e^{\alpha(T-s)}dW(s) \]

\[ Y(T) = e^{-\beta(T-t)}Y(t) + \int_t^T e^{\beta(T-s)}dL(s) \]

where \( \Lambda(t) \) is deterministic, \( W(t) \) a BM, \( L(t) \) a Lévy process.

- The forward price with delivery in \([T_1, T_2]\) is then given by

\[ F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \mathbb{E} \left[ \int_{T_1}^{T_2} S(u)du \mid \mathcal{F}_t \right] \]
Enlargement of filtrations

- We will demonstrate how to calculate the information premium in this model.
- We will enlarge the historical filtration of Lévy process $L_t$...
- ... with future information about the value $L_{T^\gamma}$.
Enlargement of filtrations

- We will demonstrate how to calculate the information premium in this model

- We will enlarge the historical filtration of Lévy process $L_t$...

- ... with future information about the value $L_{T_\gamma}$

- *Grossissemens de filtrations*:
  - Developed by French Mathematicians (Jeulin, Yor) in the 1980s
  - First theorem by Itô in 1976
Itô’s theorem for Lévy processes and additional incomplete information

**Theorem**

Let $L_t$ be a Lévy process and $\mathcal{G}_t \subseteq \mathcal{H}_t = \mathcal{F}_t \vee \sigma(L_{T_\gamma})$. Then

1. $L$ is still a semimartingale with respect to $\mathcal{G}_t$
2. If $\mathbb{E}[|L_t|] < \infty$ then

$$\xi(t) = L_t - \int_0^{t \wedge T_\gamma} \frac{\mathbb{E}[L_{T_\gamma} - L_s | \mathcal{G}_s]}{T_\gamma - s} \, ds$$

is a $\mathcal{G}_t$-martingale.
Future Lévy information

- The info premium is

\[ l_G(t, T_1, T_2; T_\gamma) = F_G(t, T_1, T_2) - F_F(t, T_1, T_2) \]

- Brownian motion terms as well as \( X_t \) and \( Y_t \) terms cancel (both filtrations coincide), thus

\[ l_G(t, T_1, T_2; T_\gamma) = \frac{1}{T_2 - T_1} \mathbb{E}\left[ \int_{T_1}^{T_2} \int_t^u e^{-\beta(u-s)} dL(s) du | G_t \right] \]

\[ - \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \phi'(0) \]

- where \( \hat{\beta} \) is some deterministic function (> 0) and \( \phi \) is the log-moment-generating function of \( L_1 \)
Future Lévy information

- We now apply Itô’s theorem

\[
\text{remember } \xi(t) = L(t) - \int_0^t \frac{\mathbb{E}[L(T_\tau) - L(s) | G_s]}{T_\tau - s} \, ds \text{ is a } G\text{-martingale)}
\]

\[
\mathbb{E} \left[ \int_{T_1}^{T_2} \int_t^u e^{-\beta(u-s)} \, dL(s) \, du | G_t \right]
\]

\[
= \mathbb{E} \left[ \int_{T_1}^{T_2} \int_t^u e^{-\beta(u-s)} \frac{\mathbb{E}[L(T_\tau) - L_s | G_s]}{T_\tau - s} \, ds \, du | G_t \right]
\]

\[
= \ldots
\]

\[
= \mathbb{E}[L_{T_\tau} - L_t | G_t] \frac{\hat{\beta}(t, T_1, T_2)}{T_\tau - t}
\]
Future Lévy information

Collecting terms yields

\[ I_G(t, T_1, T_2; T_\gamma) = \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \left( \frac{\mathbb{E}[L_{T_\gamma} - L_t | G_t]}{T_\gamma - t} - \phi'(0) \right) \]

\[ = \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \left( \mathbb{E}[L_{T_\gamma} | G_t] - \mathbb{E}[L_{T_\gamma} | \mathcal{F}_t] \right) \]
Future Lévy information

- Collecting terms yields

\[
l_G(t, T_1, T_2; T_\Upsilon) = \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \left( \frac{\mathbb{E}[L_{T_\Upsilon} - L_t | G_t]}{T_\Upsilon - t} - \phi'(0) \right)
\]

\[
= \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \left( \mathbb{E}[L_{T_\Upsilon} | G_t] - \mathbb{E}[L_{T_\Upsilon} | F_t] \right)
\]

- Sign of the premium depends on \( \mathbb{E}[L_{T_\Upsilon} | G_t] - \mathbb{E}[L_{T_\Upsilon} | F_t] \)
- ... which matches the intuition:
  - i.e. CO₂ certificates: positive premium \( \Rightarrow \)
  \[
  \mathbb{E}[L_{T_\Upsilon} | G_t] > \mathbb{E}[L_{T_\Upsilon} | F_t]
  \]
Agenda

1. Explaining the Spot-Forward Relationships
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Showing the existence of the info premium

■ Agenda:

1. Calibrate the spot model to observed data (EEX)
2. Calculate expectations under $\mathbb{P}$
3. Conduct for each class of month-forwards a constant distance-minimising change of measure (ls-sense)
4. Calculate expectation under $\mathbb{Q}$
5. Assume observed forward price $\hat{F}(t, T_1, T_2)$ is $F^Q_G(t, T_1, T_2)$
6. For the life-time of different forwards calculate

$$\hat{I}^Q_G(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F^Q_F(t, T_1, T_2)$$

(where the January 2008 forward will be our main example)
Spot Calibrating

- EEX spot from 01/02/2007 to 30/10/2008
- Includes $CO_2$-date 01/01/08 as midpoint

Figure: Spot and Simulation for data set
Expectations and change of measure

- Prices under $P$ and $Q$ and observed January 2008 forward

![Graph showing observed, $E_P$ and $E_Q$ prices over time]

**Figure:** Observed, $E_P$ and $E_Q$ Prices
The information premium?

- The residual $\hat{\mathcal{I}}_{G}(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F^Q_{F}(t, T_1, T_2)$
  - is positive approx. between 5 and 20 €
  - converges in the delivery period

Figure: The residual $\hat{\mathcal{I}}_{G}(t, T_1, T_2)$ for the January 2008 forward
Showing the existence of the info premium

- \( \hat{I}_Q^G(t, T_1, T_2) \) is our best guess for \( I(t, T_1, T_2) \)

- We need to show that:
  1. \( \hat{I}_Q^G \neq 0 \)
  2. \( \hat{I}_Q^G \) is not \( \mathcal{F}_t \)-measurable, i.e. \( \mathbb{E}[\hat{I}_Q^G | \mathcal{F}_t] = 0 \)
Showing the existence of the info premium

- \( \hat{I}^Q_G(t, T_1, T_2) \) is our best guess for \( I(t, T_1, T_2) \!

- We need to show that:
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- We will consider Nov07, Jan08, Mar08 and Aug08 contracts (lifetime before, during and after 01/01/08)
non-zero?

- $\hat{I}_G^Q \neq 0$
- All four series are clearly not white-noise
- Ljung-Box test rejected at all levels
How do we show non-measurability?

- We want to show $\mathbb{E}[\hat{I}_G | \mathcal{F}_t] = 0$
- Consider Hilbert space $L^2(\mathcal{F}, \mathbb{Q})$
- Try to express $\hat{I}_G$ in terms of a countable basis of the spot...
- ... by means of regression from $S$ onto $\hat{I}_G$
- Non-measurability $\Rightarrow$ Bad regression results!
How do we show non-measurability?

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  - Non-measurability $\Rightarrow$ Bad regression results!

- For now, let $\mathcal{B} = \{x^i : i \in \mathcal{I}\}$ the polynomial basis

- To avoid *spurious regression* (Granger/Newbold) we use (stationary) first differences
Regression results

Regression: \( \triangle \hat{I}_G(t, T_1, T_2) = \sum_{i=1}^{N} c_i \triangle S_t^i + \epsilon(t) \)
Regression results

Regression: \( \hat{\Delta I}_G(t, T_1, T_2) = \sum_{i=1}^{N} c_i \Delta S_t^i + \epsilon(t) \)

Regression results for \( N = 10 \)

<table>
<thead>
<tr>
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<th>Nov 07</th>
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<th>Mar 08</th>
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We conclude that \( \hat{\Delta I}_G(t, T_1, T_2) \) is not \( F_t \)-measurable!
Regression results

Regression: \[ \Delta \hat{I}_G^Q(t, T_1, T_2) = \sum_{i=1}^N c_i \Delta S_t^i + \epsilon(t) \]

Regression results for \( N = 10 \)

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- F-value for 95% is 1.88, thus we cannot reject \( c_1 = \ldots = c_N = 0 \)
- Increasing \( N \) does not alter the results
- Contracts living on 01/01/08 show more extreme results!
- We conclude that \( \hat{I}_G^Q(t, T_1, T_2) \) is not \( \mathcal{F}_t \)-measurable!
Discussion

- Size of $\hat{I}_G^Q$ for Jan08:
  - 2007: CO$_2$ price practically zero
  - 2008: around €22
  - assume $0.7tCO_2/MWh$ efficiency rate
  - $\Rightarrow$ info premium should be around $0.7 \cdot €22 \approx €15$
  - $\Rightarrow$ which $\hat{I}_G^Q$ is!
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  - $\Rightarrow$ which $\hat{I}_G^Q$ is!

- Other underlyings:
  - We consider electricity as an example where buy-and-hold does not work at all
  - Still, we claim that our approach is valid for other underlyings as well (unexplained risk premium!)
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- Thank you for your attention...