A State-Dependent Model for Inflation Forecasting

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Outline

A. Inflation and unemployment in recessions: Graphical evidence
B. Univariate unobserved components/stochastic volatility model
C. Multivariate extension: state-dependent Phillips Curve?
D. The current situation
Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index) (PCEPILFE) vs Civilian Unemployment Rate (UNRATE)

Shaded areas indicate US recessions. 2012 research.stlouisfed.org
Inflation and unemployment in recessions: Graphical evidence

Plots: U.S. unemployment rate and 4-quarter core inflation* during 8 NBER-dated recessions

*Core Personal Consumption Expenditure Price Index.
NBER Peak: 1969Q4

Percentage point deviation from NBER peak

Quarters after NBER peak
NBER Peak: 1980Q1

Percentage point deviation from NBER peak

Quarters after NBER peak
Unemployment and the core PCE inflation rate in U.S. recessions
Canada – averages (Jean Boivin/Bank of Canada):

**Average of Past Recession Vs Current Recession**

- **Unemployment Rate**
- **Total inflation**
- **Trend**

- **2008**
Euro area: HICP (Frank Smets)

Unemployment

-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

HICP Inflation

Scaled deviation from CEPR peak

1974Q3-1975Q1
1980Q1-1982Q3
1992Q1-1993Q3
average
2008Q1-2010Q1
Euro area: GDP deflator (Frank Smets)

Unemployment

GDP Deflator Inflation

Scaled deviation from CEPR peak

-3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5

1974Q3-1975Q1
1980Q1-1982Q3
1992Q1-1993Q3
average
2008Q1-2010Q1
Univariate unobserved components/stochastic volatility model

Stock-Watson (2007):

\[ \pi_t = \tau_t + \eta_t, \quad \eta_t = \sigma_{\eta,t} \zeta_{\eta,t}, \]

\[ \tau_t = \tau_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_{\varepsilon,t} \zeta_{\varepsilon,t}, \]

where

\[ \ln(\sigma_{\eta,t}^2) = \ln(\sigma_{\eta,t-1}^2) + \nu_{\eta,t} \]

\[ \ln(\sigma_{\varepsilon,t}^2) = \ln(\sigma_{\varepsilon,t-1}^2) + \nu_{\varepsilon,t} \]

where

\[ \begin{pmatrix} \zeta_t \\ \nu_t \end{pmatrix} \sim N \left( 0, \begin{bmatrix} I_2 & 0 \\ 0 & \gamma I_2 \end{bmatrix} \right) \]
Univariate unobserved components/stochastic volatility model, ctd.

**Approximate Moving Average (MA) representation:**

\[ \Delta \pi_t = a_t - \theta_t a_{t-1}, \quad E a_t = 0, \quad \text{var}(a_t) = \sigma_{a,t}^2 \]

\[ a_t = \pi_t - \tau_{t-1|t-1} = \text{1-step ahead forecast error} \]

The trend \( \tau_{t|t} \) is a moving average of current and past inflation:

\[ \tau_{t|t} = (1 - \theta_t) \sum_{i=0}^{\infty} \theta^i \pi_{t-i} = \begin{cases} \bar{\pi}, \theta \to 1 \\ \pi_t, \theta = 0 \end{cases} \]
Standard deviations of **permanent** and **transitory** innovations
Quarterly inflation and smoothed permanent component
Implied MA coefficient ($\theta_t$) (Core PCE)

$\theta$ large: trend inflation anchored
The UC-SV model trend inflation closely matches the Survey of Professional Forecasters (SPF) 5 year ahead median forecast.
Harvey (2008) inflation/output gap model: components

\[ \pi_t = \mu_t + \psi_t + \gamma_t + \beta_1 x_{t-1} + \beta_2 x_{t-4} + \varepsilon_t, \mu_t = \mu_{t-1} + \eta_t, \gamma_t = \text{seasonal} \]
Extension: state-dependent Phillips Curve?

Rolling RMSEs of Triangle model, relative to UCSV, v. $ugap$
Multivariate UC-SV extension: state-dependent Phillips Curve

Inflation
\[ \pi_t = \tau_t + \lambda \delta_t + \eta_t, \quad \eta_t = \sigma_{\eta,t} \xi_{\eta,t}, \]
\[ \tau_t = \tau_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_{\varepsilon,t} \xi_{\varepsilon,t}, \]

Unemployment
\[ u_t = \tilde{\tau}_t + \delta_t + \tilde{\eta}_t, \quad \tilde{\eta}_t = \sigma_{\tilde{\eta},t} \xi_{\tilde{\eta},t}, \]
\[ \tilde{\tau}_t = \tilde{\tau}_{t-1} + \tilde{\varepsilon}_t, \quad \tilde{\varepsilon}_t = \sigma_{\tilde{\varepsilon},t} \xi_{\tilde{\varepsilon},t}, \]

Common cyclical component
\[ \delta_t = \alpha_1 \delta_{t-1} + \alpha_2 \delta_{t-2} + \zeta_t, \quad \zeta_t = \sigma_{\zeta,t} \xi_{\zeta,t}, \]

Independent stochastic volatility
\[ \ln(\sigma_{\eta,t}^2) = \ln(\sigma_{\eta,t}^2) + \nu_{\eta,t}, \text{ etc. for all 5 variances} \]
\[ \left( \begin{array}{c} \xi_t \\ \nu_t \end{array} \right) \sim N \left( \begin{array}{c} 0 \\ 0 \end{array}, \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix} \right) \]

Restrictions (benchmark): \( \gamma = 0.2, \sigma_{\tilde{\varepsilon}}^2 = 0.01, \sigma_{\tilde{\eta}}^2 = 0.2 \)
Empirical results

Data
1960Q1-2009Q4, GDP deflator and total civilian unemployment rate
(a) Variance of permanent innovation to inflation

(b) Variance of transitory innovation to inflation
(a) Variance of innovation to cyclical component

(b) Variance of permanent innovation to unemployment
Posterior mean of NAIRU ($\tilde{\tau}_{l|T}$)
Posterior and prior (red) of $\alpha_1$, $\alpha_2$
Posterior and prior (red) of $\lambda$
Slope of Phillips curve: sum of coefficients in
\[ \text{Proj}(\pi_t - \tau_t \mid u_t - \tilde{\tau}_t, u_{t-1} - \tilde{\tau}_{t-1}, \ldots) \]
Table 1: Pseudo out-of-sample forecasting performance

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>h = 1</td>
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<tr>
<td>SW 2007</td>
<td>1.1375</td>
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<tr>
<td>Relative RMSFE</td>
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<tr>
<td>SW 2007</td>
<td>1.0000</td>
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<tr>
<td>AO (i)</td>
<td>1.0652</td>
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<tr>
<td>AO (ii)</td>
<td>1.0292</td>
</tr>
<tr>
<td>No Lags - $\sigma^2_\eta = 0.2$</td>
<td>0.9893</td>
</tr>
<tr>
<td>No Lags - $\sigma^2_\eta$ estimated</td>
<td>0.9894</td>
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<tr>
<td>2 Lags - $\sigma^2_\eta = 0.2$</td>
<td>0.9902</td>
</tr>
<tr>
<td>2 Lags - $\sigma^2_\eta$ estimated</td>
<td>0.9897</td>
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NOTES: Entries are RMSFEs, relative to the Stock and Watson (2007) univariate UC-SV. Bold entries are the smallest relative RMSFE for the indicated series/period/horizon.
Table 2: Pseudo out-of-sample forecasting performance

<table>
<thead>
<tr>
<th>Relative RMSFE</th>
<th>1990:I-1999:IV</th>
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<tbody>
<tr>
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<td>h = 1</td>
<td>h = 2</td>
<td>h = 4</td>
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<tr>
<td>SW 2007</td>
<td>0.5818</td>
<td>0.5019</td>
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<tr>
<td><strong>Relative RMSFE</strong></td>
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<td></td>
</tr>
<tr>
<td>SW 2007</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>AO (i)</td>
<td>0.9948</td>
<td>0.9972</td>
<td>1.0470</td>
<td>1.0278</td>
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<tr>
<td>AO (ii)</td>
<td>1.0988</td>
<td>1.0401</td>
<td>1.0470</td>
<td>1.1325</td>
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<tr>
<td>No Lags - ( \sigma^2 ) = 0.2</td>
<td>0.9555</td>
<td>0.9186</td>
<td>0.9027</td>
<td>0.9603</td>
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<tr>
<td>No Lags - ( \sigma^2 ) estimated</td>
<td>0.9547</td>
<td>0.9205</td>
<td>0.9022</td>
<td>0.9701</td>
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<tr>
<td>2 Lags - ( \sigma^2 ) = 0.2</td>
<td>0.9652</td>
<td><strong>0.9048</strong></td>
<td><strong>0.8491</strong></td>
<td><strong>0.9160</strong></td>
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<tr>
<td>2 Lags - ( \sigma^2 ) estimated</td>
<td><strong>0.9543</strong></td>
<td>0.9149</td>
<td>0.8864</td>
<td>0.9633</td>
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NOTES: see the notes in Table 1
Table 3: Pseudo out-of-sample forecasting performance

<table>
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<tr>
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<th>2000:I-2011:III</th>
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<td>h =1</td>
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<td>SW 2007</td>
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<tr>
<td>Relative RMSFE</td>
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<td>AO (i)</td>
<td>1.0104</td>
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<td>AO (ii)</td>
<td>1.1190</td>
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<tr>
<td>No Lags - $\sigma_{12}^2 = 0.2$</td>
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<tr>
<td>No Lags - $\sigma_{\eta}^2$ estimated</td>
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<tr>
<td>2 Lags - $\sigma_{12}^2 = 0.2$</td>
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<tr>
<td>2 Lags - $\sigma_{\eta}^2$ estimated</td>
<td><strong>0.9508</strong></td>
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NOTES: see the notes in Table 1
Table 4: Pseudo out-of-sample forecasting performance

<table>
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<td>SW 2007</td>
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<tr>
<td>Relative RMSFE</td>
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</tr>
<tr>
<td>SW 2007</td>
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<tr>
<td>AO (i)</td>
<td>1.0367</td>
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<tr>
<td>AO (ii)</td>
<td>1.0714</td>
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<tr>
<td>No Lags - $\sigma^2_{\eta_2} = 0.2$</td>
<td>0.9895</td>
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<tr>
<td>No Lags - $\sigma^2_{\eta_2}$ estimated</td>
<td>0.9776</td>
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<tr>
<td>2 Lags - $\sigma^2_{\eta_2} = 0.2$</td>
<td>1.0051</td>
</tr>
<tr>
<td>2 Lags - $\sigma^2_{\eta_2}$ estimated</td>
<td><strong>0.9713</strong></td>
</tr>
</tbody>
</table>

NOTES: see the notes in Table 1
Conclusions

1. Multivariate UC-SV extension captures time variation in PC slope
2. Plausible estimates of economically interpretable objects: inflation anchoring, NAIRU
3. Parsimonious
4. Some improvement in pseudo out-of-sample forecasting over univariate benchmarks