Beyond the Carry Trade: 
Optimal Currency Portfolios* 

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Abstract 

We test the relevance of technical and fundamental variables in forming currency portfolios. Carry, momentum and reversal all contribute to portfolio performance, whereas the real exchange rate and the current account do not. The resulting optimal portfolio produces out-of-sample returns that are not explained by risk and are valuable to diversified investors holding stocks and bonds. Exposure to currencies increases the Sharpe ratio of diversified portfolios by 0.5 on average, while reducing crash risk. The profitability of our optimal strategy decreases with the amount of assets under management by hedge funds, consistent with the adaptive markets hypothesis. We argue that besides risk, currency returns reflect the scarcity of speculative capital. 

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1 Introduction

Currency spot rates are nearly unpredictable out of sample (Meese and Rogoff (1983)). Usually, unpredictability is seen as evidence supporting market efficiency, but with currency spot rates it is quite the opposite – it presents a challenge. Since currencies have different interest rates, if the difference in interest rates does not forecast an offsetting depreciation, then investors can borrow the low yielding currencies to invest in the high yielding ones (Fama (1984)). This strategy, known as the carry trade, has performed extremely well for a long period without any sensible economic explanation. Burnside, Eichenbaum, and Rebelo (2008) show that a well-diversified carry trade attains a Sharpe ratio that is more than double that of the US stock market – itself a famous puzzle (Mehra and Prescott (1985)).

Considerable effort has been devoted to explaining the returns of the carry trade as compensation for risk. Lustig, Roussanov, and Verdelhan (2011a) show that the risk of carry trades across currency pairs is not completely diversifiable, so there is a systematic risk component to the strategy. They form an empirically motivated risk factor – the return of high-yielding currencies minus low-yielding currencies ($HML_{FX}$) – close in spirit to the stock market factors of Fama and French (1992) and show that it explains the carry premium. But the $HML_{FX}$ is itself a currency strategy, so linking its returns to more fundamental risk sources has been an important challenge for research in the currency market.

Some risks of the carry trade are well known. High yielding currencies are known to “go up by the stairs and down by the elevator,” implying that the carry trade has substantial crash risk. Carry performs worse when there are liquidity squeezes (Brunnermeier, Nagel, and Pederson (2008)) and increases in foreign exchange volatility (Menkhoff, Sarno, Schmeling, and Schrimpf (2012a)). Its risk exposure is time-varying, increasing in times of greater uncertainty (Christiansen, Ranaldo, and Söderlind (2011)).

Another possible explanation of the carry premium is that there is some “peso problem” with the carry trade – the negative event that justifies its returns may simply have not occurred yet. Using options to hedge away the “peso risk” reduces

abnormal returns, lending some support to this view, but the remaining returns depend crucially on the particular option strategy used for hedging (Jurek (2009)). Even so, the recent financial crisis was not the “peso event” needed to rationalize the carry trade previous returns.\footnote{Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011).}

Despite our improved understanding of the risk of the carry trade, the fact remains that conventional risk factors from the stock market (market, value, size, momentum) or consumption growth models, do not explain its returns.\footnote{Burnside, Eichenbaum, and Rebelo (2011).} Indeed, an investor looking for significant abnormal returns with respect to, say, the Fama-French factors (1992), would do very well by just dropping all equities from the portfolio and investing entirely in a passively managed currency carry portfolio instead.

Abnormal returns should not persist in a market driven by profit maximizing investors. But the currency market has a scarcity of profit-seeking capital and, conversely, an abundance of capital pursuing goals unrelated to profitability. This may explain the persistence of anomalies.

First, unlike equity markets, profit-seeking capital in currencies had a relatively minor role during most of the floating exchange rate era (Jylhä and Suominen (2011)). Second, the most important actors in the currency market – central banks – do not seek profits at all (Taylor (1982)).\footnote{When intervening, central banks stand ready to lose large amounts for extended periods of time. They typically “lean against the wind”, buying a currency that is depreciating or vice versa.}

The relevance of actors that do not maximize profits influences the profitability of speculative currency strategies. For instance, while technical analysis is close to useless in equity markets (Fama and Blume (1966)), there is considerable evidence that it produces positive risk-adjusted returns in currency markets (Levich and Thomas (1993), Taylor and Allen (1992)). LeBaron (1999) finds that the effectiveness of technical trading rules is concentrated around interventions by central banks. Silber (1994) finds similar evidence in the cross-section of currencies. So the carry trade is not the only strategy with puzzling returns in the currency market.

Market practitioners follow other approaches, including value and momentum (Levich and Pojarliev (2011)). The benefits of combining these different ap-
approaches became apparent at the height of the financial crisis when events in the currency market assumed historical proportions. Figure 1 shows the performance of three popular Deutsche Bank ETFs that track carry, value and momentum strategies with the currencies of the G10. From August 2008 to January 2009, the carry ETF experienced a severe crash of 32.6%, alongside the stock market, commodities and high yield bonds. But in the same period, the momentum ETF delivered a 29.4% return and the value ETF a 17.8% return. So while the carry trade crashed, a diversified currency strategy fared quite well in this turbulent period.

Coincidently, the literature on alternative currency investments saw major developments since 2008. Menkhoff, Sarno, Schmeling and Schrumpf (2012b) document the properties of currency momentum, Burnside (2011) examines a combination of carry and momentum, Asness, Moskowitz, and Pederson (2012) study a combination of value and momentum in currencies (and other asset classes), and Jordà and Taylor (2012) combine carry, momentum and the real exchange rate. Still, the core of the literature focuses on isolated strategies. Very few studies examine combinations of strategies and virtually none examines the optimal combination of these strategies.

Most of the studies on currency strategies focus on simple portfolios. This choice is understandable as there is substantial evidence indicating that these tend to outperform out-of-sample more complex optimized portfolios. However, this is exactly because optimized portfolios are a closer reflection of the uncertainties faced by investors in real time. Namely, they have to deal with the choice of what signals to use, how to weigh each signal, and how to address measurement error and transaction costs. This should be particularly relevant in alternative investment classes, when there is no a priori reason to believe that sorting assets by a given characteristic should produce excess returns.

To study the risk and return of currency strategies in a more realistic setting, we use the parametric portfolio policies approach of Brandt, Santa-Clara, and Valkanov (2009) and test the relevance of different variables in forming currency

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6Melvin and Taylor (2009) provide a vivid narrative of the major events in the currency market during the crisis.
7DeMiguel, Garlappi, and Uppal (2009), Jacobs, Müller, and Weber (2010).
portfolios.

First, we use a pre-sample test to study which characteristics matter for investment purposes. We test the relevance of the interest rate spread (and its sign), momentum and three proxies for value: long-term reversal, the real exchange rate, and the current account. Including all characteristics simultaneously in the test allows us to see which are relevant and which are subsumed by others. Then we conduct a comprehensive out-of-sample (OOS) exercise with 16 years of monthly returns to minimize forward-looking bias.\(^8\)

We find that the interest rate spread, momentum and reversal create economic value for investors whereas fundamentals such as the current account and the real exchange rate don’t. The strategy combining the relevant signals increases the Sharpe ratio relative to an equal-weighted carry portfolio from 0.57 to 0.86, out-of-sample and after transaction costs. This is a 0.29 gain, about the same as the Sharpe ratio of the stock market in the same period.

Transaction costs matter in currency markets. Taking transaction costs into account in the optimization further increases the Sharpe ratio to 1.06, a total gain of 0.49 over the equal-weighted carry benchmark. The gains in certainty equivalent are even more impressive as the optimal diversified strategy substantially reduces crash risk.

A set of first-pass regressions shows that the risk factors recently proposed to explain carry returns do not explain the returns of the optimized portfolio. So, while these risk factors may have some success explaining carry returns, they struggle to justify our optimal currency strategy.

Addressing a largely unexplored topic, we study the optimal combination of currency strategies with stock market factors and bonds.\(^9\) We find that including currency strategies in an optimized portfolio increases the Sharpe ratio by 0.51 on average, out-of-sample. Furthermore, adding currency strategies consistently reduces fat tails and left skewness. This contradicts crash-risk explanations for returns in the currency market.

\(^8\)Though an out-of-sample exercise does not eliminate forward looking bias completely. After all, would we be conducting the same exercise in the first place if there were no indications in the literature that momentum and value worked in recent years?

\(^9\)Kroencke, Schindler, and Schrimpf (2011) show there are benefits of investing in currencies for investors with internationally diversified holdings of stocks.
Finally, we regress the returns of the optimal strategy on the level of speculative capital in the market. We find evidence that the expected returns of the strategy decline as the amount of hedge fund capital increases. This suggests that the returns we document constitute an anomaly that is gradually being arbitraged away by hedge funds, as knowledge of the relevant currency characteristics spreads and more capital is used exploiting them—a result consistent with the adaptive markets hypothesis (Lo (2004)).

Our paper is structured as follows. In section 2 we explain the implementation of parametric portfolios of currencies. Section 3 presents the empirical analysis. Section 3.1 describes the data and the variables used in the optimization. Sections 3.2 and 3.3 present the investment performance of the optimal portfolios in and out of sample, respectively. In Section 4 we test the risk exposures of the optimal portfolio. In Section 5 we assess the value of currency strategies for investors holding stocks and bonds. Section 6 discusses possible explanations for the abnormal returns of the strategy, including insufficient speculative capital early in the sample.

2 Optimal parametric portfolios of currencies

We optimize currency portfolios from the perspective of an US investor in the forward exchange market. The investor can agree at time $t$ to buy currency $i$ forward at time $t + 1$ for $1/F_{t,t+1}^i$ where $F_{t,t+1}^i$ is the price of one USD expressed in foreign currency units (FCU). Then at time $t + 1$ the investor liquidates the position selling the currency for $1/S_{t+1}^i$, where $S_{t+1}^i$ is the spot price of one USD in FCU. The return (in USD) of a long position in currency $i$ in month $t$ is:

$$r_{t+1}^i = \frac{F_{t,t+1}^i}{S_{t+1}^i} - 1$$

This is a zero-investment strategy as it consists of positions in the forward market only.\(^{10}\) We use one-month forwards throughout as is standard in the liter-

\(^{10}\)In reality investors need to post collateral to take positions in forward markets. We ignore that in this study.
Therefore all returns are monthly and there are no inherited positions from month to month. This also avoids path-dependency when we include transaction costs in the analysis.

We optimize the currency strategies using the parametric portfolio policies approach of Brandt, Santa-Clara, and Valkanov (2009). This method models the weights of assets as a function of their characteristics. The implicit assumption is that the characteristics convey all relevant information about the assets’ conditional distribution of returns. The weight on currency \( i \) at time \( t \) is:

\[
w_{i,t} = \frac{\theta^T x_{i,t}}{N_t}
\]

where \( x_{i,t} \) is a \( k \times 1 \) vector of currency characteristics, \( \theta \) is a \( k \times 1 \) parameter vector to be estimated and \( N_t \) is the number of currencies available in the dataset at time \( t \). Dividing by \( N_t \) keeps the policy stationary (see Brandt, Santa-Clara, and Valkanov (2009)). We do not place any restriction on the weights, which can be positive or negative, reflecting the fact that in the forward exchange market there is no obvious non-negativity constraint.

The strategies we examine consist of an investment of 100% in the US risk-free asset, yielding \( r_{t}^{fUS} \), and a long-short portfolio in the forward exchange market. For a given sample, \( \theta \) uniquely determines the parametric portfolio policy, and the corresponding return each period will be:

\[
r_{p,t+1} = r_{t}^{fUS} + \sum_{i=1}^{N_t} w_{i,t} r_{t+1}^i
\]

The problem an investor faces is optimizing an objective function by picking the best possible \( \theta \) for the sample:

\[
\max_{\theta} E_t \left[ U( r_{p,t+1} ) \right]
\]

We use power utility as the objective function:

\[
U(r_p) = \frac{(1 + r_p)^{1-\gamma}}{1 - \gamma}
\]

\[11\text{Burnside, Eichenbaum, and Rebelo (2008), Burnside (2011), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Menkhoff, Sarno, Schmeling and Schrimpf (2011a,b).} \]
where $\gamma$ is the coefficient of relative risk aversion (CRRA).\textsuperscript{12} The main advantage of this utility function is that it penalizes kurtosis and skewness, as opposed to mean-variance utility which focuses only on the first two moments of the distribution of returns. So our investor dislikes crash risk and values characteristics that help reduce it, even if these do not add to the Sharpe ratio.

The main restriction imposed on the investor’s problem is that $\theta$ is kept constant across time. This substantially reduces the chances of in-sample overfitting as only a $k \times 1$ vector of characteristics is estimated. The assumption that $\theta$ does not change allows its estimation using the sample counterparts:

$$\hat{\theta} = \arg \max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} U\left(r_{t+1}^{U,S} + \sum_{i=1}^{N_t} (\theta^T x_{i,t}/N_t) r_{i,t+1}^i \right)$$

(6)

For statistical inference purposes, Brandt, Santa-Clara, and Valkanov (2009) show that we can use either the asymptotic covariance matrix of $\hat{\theta}$ or bootstrap methods.\textsuperscript{13}

For the interpretation of results it is important to note that (6) optimizes a utility function and not a measure of the distance between forecasted and realized returns. Therefore $\theta$ can be found relevant for one characteristic even if it conveys no information at all about expected returns. The characteristic may just be a predictor of a currency’s contribution to the overall skewness or kurtosis of the portfolio, for example. Conversely, a characteristic may be found irrelevant for investment purposes even if it does help in forecasting returns since it may forecast both higher returns and higher risk for a currency, offering a trade-off that is irrelevant for the investor’s utility function.

Transaction costs are relevant to assess the performance of an investment strategy (Lesmond et al. (2004)). So one valid concern is whether the gains of combining momentum with carry persist after taking into consideration transaction costs.

\textsuperscript{12}Bliss and Panigirtzoglou (2004) estimate $\gamma$ empirically from risk-aversion implicit in one-month options on the S&P and the FTSE and find a value very close to 4. We adopt this value and keep it throughout. The most important measures of economic performance of the strategy are scale-invariant (Sharpe ratio, skewness, kurtosis), so the specific choice of CRRA utility is not of crucial importance.

\textsuperscript{13}We use bootstrap methods for standard errors in the empirical part of this paper, as these are slightly more conservative and do not rely on asymptotic results.
Fortunately, parametric portfolio policies can easily incorporate transaction costs that vary across currencies and over time. This is a particularly appealing feature of the method, since transaction costs varied substantially as foreign exchange trading shifted towards electronic crossing networks.

To address this issue we optimize:

\[
\hat{\theta} = \arg \max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} U \left( r_{t}^{US} + \sum_{i=1}^{N_i} (\theta^T x_{i,t}/N_i) r_{i,t+1}^{i} - \sum_{i=1}^{N_i} |\theta^T x_{i,t}/N_i| c_{i,t} \right)
\]

where \(c_{i,t}\) is the transaction cost of currency \(i\) at time \(t\), which we calculate as:

\[
c_{i,t} = \frac{F_{\text{ask}}^{i,t,t+1} - F_{\text{bid}}^{i,t,t+1}}{F_{\text{ask}}^{i,t,t+1} + F_{\text{bid}}^{i,t,t+1}}
\]

This is one half of the bid-ask spread as a percentage of the mid-quote. This assumes the investor buys (sells) a currency in the forward market at the ask (bid) price, and the forward is settled at the next month’s spot rate.\(^{14}\)

For a given month and currency, transaction costs are proportional to the absolute weight put on that particular currency. This absolute weight is a function of all the currency characteristics as seen in equation 2, so transaction costs depend crucially on the time-varying interaction between characteristics. One example is the interaction between momentum and other characteristics. As Grundy and Martin (2001) show for stocks, the way momentum portfolios are built guarantees time-varying interaction with other stock characteristics. For instance, after a bear market, winners tend to be low-beta stocks and the reverse for losers. So the momentum portfolio, long in previous winners and short in previous losers, will have a negative beta. The opposite holds after a bull market. The same applies for currencies, after a period when carry experiences high returns, high yielding currencies tend to have positive momentum. In this case, momentum reinforces the carry signal and results in larger absolute weights and thus higher transaction costs. However, after negative carry returns the opposite happens: high yielding currencies have negative momentum. So momentum partially offsets

\(^{14}\) Actually, this may overstate transaction costs. For instance, Mancini, Ranaldo, and Wrampelmeyer (2011) document that effective costs in the spot market are less than half those implied by bid-ask quotes as there is significant within-quote trading.
the carry signal resulting in smaller absolute weights and actually reduces the overall transaction costs of the portfolio. This means the transaction costs of including momentum for an extended period of time in a diversified portfolio policy will be lower than what one finds examining momentum in isolation as in Menkhoff, Sarno, Schmeling, and Schrimpf (2012b).

3 Empirical analysis

As figure 1 shows, combining reversal and momentum with the carry trade considerably mitigated the crash of the carry trade in the last quarter of 2008. Yet this is easy to point out \textit{ex post}. The relevant question is whether investors in the currency market had reasons to believe in the virtue of diversifying their investment strategy before the 2008 crash. For example, Levich and Pojarliev (2011) examine a sample of currency managers and find that they explored carry, momentum and value strategies before the crisis but shifted substantially across investment styles over time. In particular, right before the height of the financial crisis in the last quarter of 2008, most currency managers were heavily exposed to the carry trade, neutral on momentum and investing \textit{against} value. This raises the question of whether the benefits of diversification were as clear before the crisis as they later became apparent.

To address this issue we conduct two tests: i) a pre-sample test with the first 20 years of data up to 1996 to determine which characteristics were relevant at that time; ii) an out-of-sample experiment since 1996 in which the investor chooses the weight to put on each signal using only historical information available up to each moment in time.

Section 3.1. explains the data sources and the variables used in our optimization. In section 3.2. we conduct the pre-sample test with the sample from 1976:02 to 1996:02. In section 3.3. we conduct the out-of-sample experiment of portfolio optimization using only the relevant variables identified in the pre-sample test.
3.1 Data

We use data on exchange rates, the forward premium, and the real exchange rate for the Euro zone and the 27 member countries of the Organization for Economic Cooperation and Development (OECD). The countries in the sample are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, South Korea, Spain, Sweden, Switzerland, the UK, and the US.

Most studies in the recent literature on currency returns use broader samples of countries, including many emerging economies (e.g., Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Lustig, Roussanov, and Verdelhan (2011a,b)). But this raises possible issues of selection bias. It may also be hard to point out the exact time when an emerging country currency first became an eligible asset to invest. To avoid these issues, we restrict ourselves to OECD members—the “developed countries club.”

The exchange rate data are from Datastream. They include spot exchange rates at monthly frequency from November 1960 to December 2011 and one-month forward exchange rates from February 1976. As in Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) we merge two datasets of forward exchange rates (against the USD and the GBP) to have a comprehensive sample of returns in the forward market in the floating exchange rate era.

We calculate the real exchange rates of each currency against the USD using the spot exchange rates and the consumer price index. The Consumer Price Index (CPI) data come from the OECD/Main Economic Indicators (MEI) online database. In the case of Australia, New Zealand, and Ireland (before November 1975) only quarterly data are available. In those cases, the value of the last available period was carried forward to the next month. In the case of the Euro, we use the Harmonised Index of Consumer Prices (HICP) from the European Central Bank.

\[15\] We also see no reason to restrict the sample further to just the three major currencies (as some studies do) or even the G10. The assets we consider were perfectly eligible to invest and a portfolio optimization will be of little interest if the universe of assets becomes too small.

\[16\] The first dataset has data on forward exchange rates (bid and ask quotes) against the GBP from 1976 to 1996 and the second dataset has the same information for quotes against the USD from 1996 to 2011.
instead. The series that starts in January 1996, was extended back to January 1988 using the weights in the HICP of the Euro founding members.

We test the economic relevance of carry, momentum, value proxies combined with fundamentals in a currency market investment strategy. The variables used in the optimization exercise are:

1. $\text{sign}_{i,t}$: The sign of the forward discount of a currency with respect to the USD. It is 1 if the foreign currency is trading at a discount ($F_{i,t} > S_{i,t}$) and -1 if it trades at a premium. This is the carry trade strategy examined in Burnside, Eichenbaum, and Rebelo (2008), Burnside (2011), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011). Given the extensive study of this strategy we adopt it as the benchmark throughout the paper.

2. $\text{fd}_{i,t}$: The interest rate spread or the forward discount on the currency. We standardize the forward discount using the cross-section mean and standard deviation across all countries available at time $t$, $\mu_{FD_t}$ and $\sigma_{FD_t}$ respectively. Specifically, denoting the (unstandardized) forward discount as $FD_{i,t}$, we obtain the standardized discount as: $fd_{i,t} = \frac{FD_{i,t} - \mu_{FD_t}}{\sigma_{FD_t}}$. This cross-sectional standardization measures the forward discount in standard deviations above or below the average across all countries. By construction, a variable standardized in the cross-section will have zero mean, implying that the strategy is neutral in terms of the base currency (the US dollar).\footnote{Standardizing characteristics in the cross-section of assets is a usual first step in the construction of parametric portfolio policies (although not a pre-requisite of the method). See Brandt, Santa-Clara, and Valkanov (2009).}

3. $\text{mom}_{i,t}$: For currency momentum we use the cumulative currency appreciation in the last three-month period, cross-sectionally standardized. This variable explores the short-term persistence in currency returns. We use momentum in the previous three months because there is ample evidence for persistence in returns for portfolios with this formation period while there are no significant gains (in fact the momentum effect is often smaller) considering longer formation periods (see Menkhoff, Sarno, Schmeling, and Schrimpf (2012b)). Three-month momentum was also used in Kroencke, Schindler,
and Schrimpf (2011). Cross-sectional standardizations means that momentum measures relative performance. Even if all currencies fall relative to the USD those that fall less will have positive momentum.

4. \( \text{rev}_{i,t} \): Long-term reversal is the cumulative real currency depreciation in the previous five years, standardized cross-sectionally. First we calculate the cumulative real depreciation of currency \( i \) between the basis period \((h)\) and moment \( t \) as an index number \( Q_{i,h,t} = \frac{S_{i,t} \text{CPI}_{i,t-2}\text{CPI}_{h-2}^{1/2}}{S_{i,h} \text{CPI}_{i,t-2} \text{CPI}_{h-2}^{1/2}} \). We use a two-month lag to ensure the CPI is known. We pick \( h = t - 60 \) which corresponds to 5 years. Then we standardize \( Q_{i,h,t} \) cross-sectionally to obtain \( \text{rev}_{i,t} \). This is essentially the same as the notion of “currency value” used in Asness, Moskowitz, and Pederson (2012). We just use the cumulative deviation from purchasing power parity, instead of the cumulative return as they did, to obtain a longer out-of-sample test period. Reversal is positive for those currencies that experienced the larger real depreciations against the USD in the previous 5 years and negative for the others.

5. \( \text{q}_{i,t} \): The real exchange rate standardized by its historical mean and standard deviation. As for reversal, we compute \( Q_{i,h_{i},t} \) with the difference that here the basis period \((h_{i})\) is the first month for which there is CPI and exchange rate data available for currency \( i \). Then we compute \( \text{q}_{i,t} = \frac{Q_{i,h_{i},t}-\overline{Q}_{i,t}}{\sigma Q_{i,t}} \), where \( \overline{Q}_{i,t} \) is the historical average \( \sum_{j=h_{i}}^{t} Q_{i,h_{i},j}/t \) and \( \sigma Q_{i,t} \) is the historical standard deviation \( \sigma (\{Q_{i,h_{i},j}\}_{j=h_{i}}^{t}) \). The real exchange rate is measured in standard deviations above or below the historical average. Historical standardization is needed as the real exchange rate is very close to a unit root process. As such the average distance from the historical mean each moment in time depends on the number of previous observations in sample.\(^{18} \) Historical standardization ensures the optimization does not overweight the signal for currencies with longer samples. Unlike \( \text{rev} \), which is cross-sectionally standardized, \( q \) is not neutral in terms of the basis currency (the USD). It will tend to be positive for all currencies when these are undervalued against the USD by

\(^{18}\)This is not the same for every currency as for some data starts at different periods than it does for others.
historical standards.

6. \(\text{ca}_{t,t-1}\): The current account of the foreign economy as a percentage of Gross Domestic Product (GDP), standardized cross-sectionally. The optimization assumes that the previous year current account information becomes known in April of the current year. The current account data were retrieved from the Annual Macroeconomic database of the European Commission (AMECO), where data are available on a yearly frequency from 1960 onward. Many studies examine the relation between the current account and exchange rates justifying its inclusion as a conditional variable.\(^{19}\)

In order to be considered for the trading strategies, a currency must satisfy three criteria: i) there must be at least ten previous years of real exchange rate data; ii) current forward and spot exchange quotes must be available; and iii) the country must be already an OECD member in the period considered. After filtering out missing observations, there are a minimum of 13 and a maximum of 21 currencies in the sample. On average there are 16 currencies in the sample at each point in time.

### 3.2 Pre-sample results

Table 1 shows the investment performance of the optimized strategies from 1976:02 to 1996:02. We use this pre-sample period to check which variables had strong enough evidence supporting their relevance back in 1996, before starting the out-of-sample experiment.

The two versions of the carry trade (\(\text{sign}\) and \(\text{fd}\)) deliver similar performance, with high Sharpe ratios (0.96 and 0.99, respectively) but also with significant crash risk (as captured by excess kurtosis and left-skewness). Momentum provides a Sharpe ratio of 0.56, better than the performance of the stock market of 0.40 in the same sample. Okunev and White (2003), Burnside, Eichenbaum and Rebelo (2011), and Menkhoff, Sarno, Schmeling, and Schrimpf (2012b) all document the presence of momentum in currency markets.

\(^{19}\)See, for example, Dornbusch and Fischer (1980), Obstfeld and Rogoff (2005), Gourinchas and Rey (2007).
Financial predictors work better in our optimization than fundamentals like the real exchange rate and the current account. Reversal had an interesting Sharpe ratio of 0.36.\textsuperscript{20} However, the strategies using the current account and the real exchange rate as conditioning variables achieved modest Sharpe ratios (of 0.16 and 0.07), not at all impressive – especially as this is an in-sample optimization.\textsuperscript{21}

The seventh row shows the performance of an optimal strategy combining the carry (both \textit{sign} and \textit{fd}) with momentum and reversal – all the statistically relevant variables. Already in 1996 there was ample evidence indicating that a strategy combining different variables leads to substantial gains. The Sharpe ratio of the optimal strategy was nearly 40\% higher than the benchmark and it produced a 16.43 percentage points gain in annual certainty equivalent.

Adding fundamentals to this strategy does not improve it: the Sharpe ratio increases only 0.01 and the annual certainty equivalent only 13 basis points. An insignificant gain since in-sample any additional variable must always increase utility.

Table 2 shows the statistical significance of the variables, isolated and in combination. The table presents the point-estimates of the coefficients and the bootstrapped p-values (in brackets). We perform the bootstrap by generating 1,000 random samples drawn with replacement from the original sample and with the same number of observations (240 months of returns and respective conditional variables). Then we find the optimal coefficients in each random sample, thereby obtaining their distribution across samples.

Taken in isolation, the carry trade variables (\textit{sign} and \textit{fd}) and momentum are all significant at the 1\% level. Reversal has a p-value of 5.3\%.

The current account and the real exchange rate have the wrong sign (underweighting undervalued currencies and those with strong current accounts) but these signs are not significant. We have known since Meese and Rogoff (1983) that currency spot rates are nearly unpredictable by fundamentals, a result known as the

\textsuperscript{20}Reversal is similar to the real exchange rate but it throws away the data with more than 5 years each moment in time. We believe this is its crucial advantage in a sample where real exchange rates are not available for all currencies and for all periods simultaneously.

\textsuperscript{21}We also tested these variables out-of-sample (although, based on the in-sample evidence, the investor would choose not to consider them) and found that they did not add to the economic value of the strategy.
“disconnect puzzle.” Gourinchas and Rey (2007) find that the current account forecasts the spot exchange rate of the US dollar against a basket of currencies. But we find no evidence in the cross section that the current account is relevant at all for designing a profitable portfolio of currencies. This does not imply that fundamentals have no effect on exchange rates. Only that expectations about future fundamentals are already embodied in present spot rates (see Engel and West (2005) and Sarno and Schmeling (2012)), so that fundamental variables are subsumed by technical variables.

Combining all variables confirms our main result. Carry, momentum and reversal are relevant for the optimization, fundamentals are not. The final row shows the results for an optimization using only the variables deemed relevant. The p-values show that the four variables contribute significantly to the economic value of the strategy in combination.

Concerning both carry variables (sign and $fd$), the correlation of their returns was only 0.46 from 1976:02 to 1996:02, a value that has not changed much since. So these two ways of implementing the carry trade are not identical and the investor finds it optimal to use both. The sign variable assigns the same weight to a currency yielding 0.1% more than the USD as to another yielding 5% more. In contrast, the $fd$ variable assigns weights proportionally to the magnitude of the interest rate differential. Whenever the USD interest rate is close to the extremes of cross section, the sign is very exposed to variations in its value, while $fd$ is always dollar-neutral.

One word of caution on forward-looking bias is needed here. Our pre-sample test shows that as of 1996 some of the strategies recently proposed in the literature on currency returns would already be found to have an interesting performance. This is a necessary condition to assess if investors would want to use these variables in real time to build diversified currency portfolios. However, this does not tell us whether there were other investment variables that we do not test that would have seemed relevant in 1996 and resulted afterwards in poor economic performance.

---

22 Gourinchas and Rey (2007) derive their result making a different use of the current account information. Namely, they detrend it and also consider net foreign wealth.
3.3 Out-of-sample results

We perform an out-of-sample (OOS) experiment to test the robustness of the optimal portfolio combining carry, momentum, and value strategies. The first optimal parametric portfolio is estimated using the initial 240 months of the sample. Then the model is re-estimated every month, using an expanding window of data until the end of the sample. The out-of-sample returns thus obtained minimize the problem of look-ahead bias. We do not use \( q \) and \( ca \) in the optimization as these failed to pass the in-sample test with data until 1996.\(^{23}\)

The in-sample results also hold out of sample. Table 3 shows that the model using interest rate variables, momentum and reversal achieves a certainty equivalent gain of 10.84 percent over the benchmark, with better kurtosis and skewness. Its Sharpe ratio is 1.15, a gain of 0.45 over the benchmark \( \text{sign} \) portfolio.

Transaction costs can considerably hamper the performance of an investment strategy. For example, Jegadeesh and Titman (1993) provide compelling evidence that there is momentum in stock prices, but Lesmond et al. (2004) find that after taking transaction costs into consideration there are little to no gains to be obtained in exploiting momentum.

Panel B of table 3 shows the OOS performance of the strategies after taking transaction costs into consideration. Clearly transaction costs matter. The Sharpe ratio of the optimal strategy is reduced by 0.29, a magnitude similar to the equity premium, and the certainty equivalent drops from 18.87 percent to just 12.15 percent. Momentum and reversal individually show no profitability at all after transaction costs. This suggests a simple explanation for the new evidence on currency return predictability: investors could not exploit it due to transaction costs, hence its persistence. Unfortunately this explanation does not hold.

In our perspective, measuring the transaction costs of individual currency strategies, as often done in the literature, is inadequate and overstates the importance of transaction costs altogether. For example, say the stand-alone momentum

\(^{23}\)Although including fundamentals does not change much the results as they receive little weight in the optimization. Based on the p-value of reversal (significant at 10% but not at 5%), we might question whether the investor would have wanted to use it in the out-of-sample period. But the coefficient shows the correct sign and there were many indications in the literature that reversal worked for other assets too. So we include reversal in the out-of-sample test.
strategy is not profitable after transaction costs, but a carry strategy is so. Then the investor will want to follow the carry strategy. The relevant problem for the investor is not whether stand-alone momentum is exploitable after trading costs but rather if using momentum on top of carry is beneficial after the increase in total transaction costs it implies. In practice, the momentum signal reinforces the carry signal for some currency-periods, resulting in higher trading costs, but momentum offsets carry for other currency-periods, decreasing transaction costs. A priori there is no way of telling if a high-cost stand-alone strategy, such as momentum, actually results in increased costs for the investor. All depends on the interaction between signals. The final row of panel B illustrates our point. The strategy using all signals (even those that do not produce value individually after transaction costs) still results in substantial outperformance, increasing the certainty equivalent relative to the benchmark by 5.56 percentage points.\footnote{A strategy using only \( \text{fd} \) and \( \text{sign} \) achieves OOS a certainty equivalent of 6.21 percentage points. Hence momentum and reversal add value to the portfolio even after transaction costs.}

Furthermore, we find that transaction costs can be managed. In panel C we adjust the optimization to currency and time-specific transaction costs. We calculate a cost-adjusted interest rate spread variable: 
\[ \hat{F}D_{i,t} = \text{sign}(FD_{it})(|FD_{it}| - c_{it}) \]
and standardize it in the cross-section to get \( \hat{f}_{d_{it}} \). We use this variable instead of \( f_{d_{i,t}} \) in the vector of currency characteristics \( x_{i,t} \). We then model the parametric weight function as:
\[ w_{i,t} = I(c_{it} < |FD_{it}|) \left[ \theta^T x_{i,t}/N_t \right] \]  
where \( I(\cdot) \) is the indicator function, with a value of one if the condition holds and zero otherwise. We maximize expected utility with this new portfolio policy, estimating \( \theta \) after consideration of transaction costs.

This method effectively eliminates from the sample currencies with prohibitive transaction costs and reduces the exposure to those that have a high ratio of cost to forward discount. Other, more complex, rules might lead to better results, but we refrain from this pursuit as this simple approach is enough to prove the point that managing transaction costs adds considerable value.

The procedure increases the Sharpe ratio of the diversified strategy from 0.86 to 1.06 and produces a gain in the certainty equivalent of 4.54 percent per year.
This gain alone is higher than the momentum or reversal certainty equivalents per se. Indeed, the performance of the diversified strategy with managed transaction costs is very close to the strategy in panel A without transaction costs.

Managing transaction costs is particularly important as these currency strategies are leveraged. Given the high Sharpe ratios attainable by investing in currencies, the optimization picks high levels of leverage. We define leverage as

$$L_t = \sum_{i=1}^{N_i} |w_{it}|.$$  

This is the absolute value risked in the currency strategy per dollar invested in the risk-free asset. The optimal strategy has a mean leverage of 5.94 in the OOS period of 1996:03 to 2011:12. This means, that for each dollar invested in the risk free rate, the investor would be long 3-dollars worth of some set of foreign currencies and short 3-dollars worth of another set of currencies, approximately. As a result, a small difference in transaction costs can have a large impact in the economic performance of the strategy.

One concern in optimized portfolios is whether in-sample overfitting leads to unstable and erratic coefficients OOS. Figure 2 shows the estimated coefficients of the diversified portfolio with managed costs in the OOS period. The coefficients of the four variables used are stable, leading to consistent exposure to the conditioning variables.

The optimal diversified portfolio has a robust OOS economic performance. In fact, while currency managers shift erratically across styles, without showing any particular timing hability (Levich and Pojarliev (2011)), our strategy does not suffer from the same problem.\(^{25}\)

## 4 Risk exposures

Cochrane (2011) uses the expression “factor zoo” to describe the growing number of risk factors proposed in the literature to explain asset returns. The literature on currency markets is no exception and many sets of risk factors have been proposed, mostly to explain the returns of the carry trade.

Lustig, Roussanov, and Verdelhan (2011a) propose an empirically-motivated

\(^{25}\)In fact, some practitioners shared with us that what they are really interested in finding is a better method to shift across styles. For now our advice is simple: don’t!
high-minus-low factor of currencies sorted on interest rates ($HML_{FX}$) to explain carry trade returns. This is an approach similar in spirit to the Fama and French (1992) three-factor model for stock returns. Note however that the $HML_{FX}$ factor is itself by construction a carry portfolio. So while this approach establishes that there is systematic risk in the carry trade, it does not provide intuition on what is the fundamental risk source that justifies its returns. Brunnermeier, Nagel, and Pederson (2008) argue that liquidity-risk spirals are the source of risk for the carry trade. They use the innovation in the TED spread and in the VIX as factors proxying for liquidity and risk. Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) propose innovations in foreign exchange market volatility as a risk factor to explain the carry trade and currency momentum. They also use the innovation in average transaction costs and argue the information in this is subsumed by FX volatility. Lustig and Verdelhan (2007) and Lustig, Roussanov, and Verdelhan (2011b) propose consumption growth risk as a factor to explain the carry returns. Table 4 shows the exposure of the optimal diversified strategy (with managed transaction costs) to 8 sets of risk factors.

The first model shows that the currency strategy is not exposed to consumption growth risk.\footnote{For this we use the monthly growth rate of Real Personal Consumption Expenditures downloaded from the Federal Reserve of St. Louis.} Burnside (2012) finds a similar result for the carry trade, one of the elements used in our strategy.

The second and third models show that our strategy is exposed to liquidity risk (as captured by innovations in the TED spread) and increases in stock volatility (as captured by the changes in VIX). The VIX is a more significant variable, its beta has a t-statistic of -3.98 versus -2.90 for the TED spread.

The fourth model regresses the returns of the optimal strategy on innovations in transaction costs (the cross-sectional average in the forward exchange market). This does not yield significant results as the adjusted R-squared is negative.

The fifth model shows that the diversified portfolio, with a t-statistic of -2.15, is exposed to innovations in foreign exchange volatility as Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) find for the carry trade.\footnote{We follow Menkhoff, Sarno, Schmeling, and Schrimpf (2011a) in computing FX volatility in month $t$ as: $\sigma_{FX,t} = \frac{1}{D_t} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} \frac{|e_{it}|}{N\tau}$, where $D_t$ is the number of trading days in month $t$ and $N\tau$ is the number of observations in month $t$.} But the adjusted
R-squared is only 1.88%.

Our optimal strategy is also somewhat exposed to stock market risk as the CAPM and the Carhart (1997) models show. But the only relevant variable is the excess return on the market portfolio with a t-statistic of 4.02 in the CAPM and 4.08 in the Carhart four-factor model.

The best performing model, in term of adjusted R-squared, is the empirically-motivated \( HML_{FX} \) factor of Lustig, Roussanov, and Verdelhan (2011a). In this model we regress the optimal portfolio excess returns on \( RX \) (the dollar-return of an equal-weighted average of all currencies) and \( HML_{FX} \), the difference in return between the highest yielding currencies and the lowest yielding currencies.\(^{28}\) The beta with respect to \( HML_{FX} \) is clearly significant, with a t-stat of 6.54, and the adjusted R-squared of 20.85 is by far the highest among the eight models used.

For the last three regressions, that use investable factors as regressors, the most striking result is the high \( \alpha \) of the optimal strategy, ranging between 1.73 and 2.19 percent per month. This is close to the mean monthly return of the strategy. So, while the optimal strategy is exposed to some of the factors proposed in the literature on currency returns, its risk-adjusted returns is highly significant and almost identical to the unadjusted return.

There is evidence of time-varying risk exposures in the carry trade (Christiansen, Ranaldo, and Söderlind (2011)). In particular, the exposure of the carry to the stock market rises after shocks to liquidity and risk. This is not captured by the unconditional analysis in table 4. So it is of interest to ask whether our optimal strategy also has time-varying risk.

Following Christiansen, Ranaldo, and Söderlind (2011) we run the following OLS regression:

\[
r_{p,t} - r_f = \alpha + \beta_0 R_{MRF_t} + \beta_1 R_{MRF_t} z_{t-1} + \beta_2 R_{bonds,t} + \beta_3 R_{bonds,t} z_{t-1} + \epsilon_t \quad (10)
\]

where \( z_{t-1} \) is a proxy for (lagged) risk and \( R_{bonds,t} \) is the excess return of the 10 year US bond over the risk-free rate.\(^{29}\) As proxies for risk we use the foreign exchange

\(^{28}\)We retrieve the data from Adrien Verdelhan’s webpage. The data is for returns with all currencies and after transaction costs.

\(^{29}\)Bond returns are from Datastream.
volatility, the TED spread, VIX, the average transaction cost, and leverage. The first four are used in Christiansen, Ranaldo, and Söderlind (2011). We add leverage as this is time varying in the optimal strategy and could naturally induce time-varying risk.

The results of the regression are in table 5. The only interaction term that is significant is for the TED spread with the market. But the sign of the coefficient is negative, implying the strategy is less exposed to the stock market after a liquidity squeeze. In order for time-varying risk to explain the returns of the diversified strategy, the opposite should happen. All other interaction terms are not significant, so time-varying risk is of little relevance to explain the performance of the diversified strategy. In particular, there is no evidence that the optimal strategy is riskier when it is more leveraged. In general, the conditional models do not add much to the CAPM, and the large significant $\alpha$ persists after considering time-varying risk.

Either unconditionally or conditionally the risk factors proposed to explain the carry trade can do very little to explain the returns of our optimal diversified currency strategy. This indicates that the optimal strategy exploits market inefficiencies rather than loading on factor risk premiums.

5 Value to diversified investors

We assess whether the currency strategies are relevant for investors already exposed to the major asset classes. Indeed, there is no reason a priori that investors should restrict themselves to pure currency strategies, particularly when there are other risk factors that have consistently offered significant premiums as well.

The value of currency strategies to diversified investors holding bonds and stocks is a relatively unexplored topic. Most of the literature on the currency market has focused on currency-specific strategies.

We continue to assume that the investor optimizes power utility with constant relative risk aversion of 4. The returns on wealth are now:

$$ R_{p,t+1} = r^{US}_t + \sum_{j=1}^{M} w_j F_j + \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} - \sum_{i=1}^{N_t} |w_{i,t}| c_{i,t} $$

(11)
where $w_j$ are the (constant) weights on a set of $M$ investable factors $F$ expressed as excess returns, and $w_{it}$ depends on the characteristics and the $\theta$ coefficients that maximize utility jointly with $w_j$.

Table 6 shows the OOS performance of the portfolios with and without the currency strategy. The currency strategy combines the interest rate spread, sign, momentum, and long-term reversal. Subsequently, each two rows compare a portfolio of investable factors with a portfolio combining these factors with the currency strategy.

The opportunity to invest in currencies is clearly valuable to investors. Including currencies in the portfolio always adds to the Sharpe ratio and raises the certainty equivalent. The OOS gains in certainty equivalent range between 9.99 percentage points for an investment in stocks and bonds and 38.04 percentage points for a diversified investment using the Carhart factors. The gain with respect to the Carhart factors comes mainly from the dismal performance of stock momentum in 2009, when it experienced one of its worst crashes in history (Daniel and Moskowitz (2012), Barroso and Santa-Clara (2012)).

These gains are far more impressive than the gains from adding factors like HML and SMB to the stock market. Indeed, only the inclusion of bonds improves upon the certainty equivalent of the stock market OOS. Generally, the inclusion of SMB, HML, and WML factors improves Sharpe ratios, but this increase is offset by higher drawdowns, resulting in lower certainty equivalents.

Including currencies however leads to substantial gains. The relevance of the interest rate spread, currency momentum, and long-term reversal to forecast currency returns makes all conventional risk premiums seem small in comparison.

Including currencies in the portfolio of stocks and bonds produces increases in the Sharpe ratio as high as 0.81 for a portfolio of US stocks and currencies. On average adding currency strategies increases the Sharpe ratio by 0.51.

One possible justification for the higher Sharpe ratios obtainable by investing in currencies is that these might entail a higher crash risk – as Brunnermeier, Nagel, and Pedersen (2008) shows for the carry trade. But diversified currency strategies do not conform to this explanation. Figure 3 shows how complementing a portfolio policy with investments in the currency market reduces substantially the excess kurtosis and left-skewness of diversified portfolios.
Our results make it hard to reconcile the economic value of currency investing with the existence of some set of risk factors that drives returns in currencies and other asset classes. The substantial increases in Sharpe ratios combined with the lower crash risk indicate that there is either a specific set of risk factors in the currency market or that currency returns have been anomalous throughout our sample.

6 Speculative capital

We cannot justify the profitability of our currency strategy as compensation for risk. The obvious alternative explanation is market inefficiency. This might persist due to insufficient arbitrage capital, possibly because strategies exploring the cross section of currency returns were not well known. This argument is consistent with the adaptive markets hypothesis of Lo (2004). This hypothesis argues that it takes time for arbitrageurs to gather enough capital to fully exploit one source of anomalous risk-adjusted returns. As such, an anomaly can persist for some time, even if not indefinitely.

Jylhä and Suominen (2011) find that carry returns explain hedge fund returns even after controlling for the other factors proposed by Fung and Hsieh (2004) and that growth in hedge fund speculative capital is driving carry trade profits down. Neely, Weller, and Ulrich (2009) document a similar decline over time of the profitability of technical trading analysis rules in the currency market.

We run an OLS regression of the returns of the optimal strategy on hedge fund assets under management scaled by the monetary aggregate M2 of the 11 currencies in their sample \((\frac{AUM}{M2})\) and new fund flows \((\Delta \frac{AUM}{M2})\).\(^{30}\) The regression uses the out-of-sample returns, after transaction costs, of the optimal strategy from 1996:03 to 2008:12 as the dependent variable. The estimated coefficients (and t-statistics in parenthesis) are:

\[
\begin{align*}
    r_{p,t} &= 0.08 - 1.47 \left( \frac{AUM}{M2} \right)_{t-1} + 3.56 \left( \Delta \frac{AUM}{M2} \right)_t \\
    &\quad (4.29) \quad (-3.23) \quad (0.36)
\end{align*}
\]

The R-squared of the regression is 6.5%. The new flow of capital to hedge

\(^{30}\text{We thank Matti Suominen for providing us the time series of AUM/M2. See their paper for a more detailed description of the data.}\)
funds is not significant in the regression but the estimated coefficient has the correct sign. The level of hedge fund capital predicts negatively the returns of the optimal strategy. With a t-statistic of -3.23, this provides convincing evidence that the returns of the diversified currency strategy are an anomaly that is gradually being corrected as more hedge fund capital exploits it.\footnote{The significance of the coefficient of AUM/M2 is robust to the inclusion of a time variable in the right hand side. So this result can not be attributed to a mere trend effect.} This result supports the adaptive markets hypothesis in the currency market and complement the existing evidence of Neely, Weller, and Ulrich (2009) and Jylhä and Suominen (2011).

This opens the question of whether the large returns of the strategy are likely to continue going forward. We note that in the last three years of our sample (2009-2011) the strategy produces a Sharpe ratio of 0.82, lower than its historical average but still an impressive performance (though not much different than the stock market in the same period).

7 Conclusion

Diversified currency investments using the information of momentum, yield differential, and reversal, outperform the carry trade substantially. This outperformance materializes in a higher Sharpe ratio and in less severe drawdowns, as reversal and momentum had large positive returns when the carry trade crashed. The performance of our optimal currency strategy poses a problem to peso explanations of currency returns.

Our optimal currency portfolio picks stable coefficients for the relevant currency characteristics and adds more value by dealing with transaction costs.

The economic performance of the optimal currency portfolio cannot be explained by risk factors or time-varying risk. This suggests market inefficiency or, at least, that the right risk factors to explain currency momentum and reversal returns have not been identified yet.

Investing in currencies significantly improves the performance of diversified portfolios already exposed to stocks and bonds. So currencies either offer exposure to some set of unknown risk factors or have anomalous returns.

The most convincing explanation for the returns of our optimal diversified
currency portfolio is that it constitutes an anomaly – one which is being gradually arbitraged away as speculative capital increases in the foreign exchange market. This is consistent with the adaptive markets hypothesis of Lo (2004).

By using new optimization technology on old currency data, we show that the puzzles in the currency market are too deep (and the economic performance of the resulting strategy too impressive) to support a risk-based explanation.

References


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**Table 1.** The in-sample performance of the investment strategies in the period 1976:02 to 1996:02. The optimizations use a power utility with CRRA of 4. The max and the min are, respectively, the maximum and the minimum one-month return in the sample, expressed in percentage points. The mean is the annualized average return, in percentage points. The standard deviation and Sharpe ratio are also annualized and “Kurt.” stands for excess kurtosis. The certainty equivalent is expressed in annual percentage points. It is the constant return that would provide the same utility as the series of returns of the given strategy. The first 6 rows show the results for a strategy based on using only one variable at a time. The seventh row shows the results for a strategy combining the four relevant signals. The last row shows the performance of a strategy combining all variables simultaneously. See description of the variables in the text.
Table 2. The statistical significance of the variables in the in-sample period of 1976:02 to 1996:02. The coefficient estimates and bootstrapped p-values (in brackets). The coefficient estimates are the ones that maximize in-sample a power utility function over wealth with a CRRA of 4. To obtain the p-values we generate 1000 random samples of the same size as the original sample, drawing each observation with replacement. Then for each sample we re-estimate the optimal coefficient. The p-value is the percentage of random samples where the estimated coefficient has a sign opposite to the expected.
### Table 3.
The OOS performance of the investment strategies in the period 1996:03 to 2011:12 with different methods to deal with transaction costs. Panel A presents the results without considering transaction costs. Panel B takes transaction costs into consideration. Panel C excludes all currencies whenever the bid-ask spread is higher than the forward discount, then adjusts the forward discount by the transaction cost. All optimizations use a power utility function with a CRRA of 4 and the coefficients are re-estimated each month using an expanding window of observations in the OOS period of 1996:03 to 2011:12. The max and the min are, respectively, the maximum and the minimum one-month return in the sample, expressed in percentage points. The mean is the annualized average return, in percentage points. The standard deviation and Sharpe ratio are also annualized and “Kurt.” stands for excess kurtosis. The certainty equivalent is expressed in annual percentage points. It is the constant return that would provide the same utility as the series of returns of the given strategy.

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<td>15.70</td>
<td>2.14</td>
<td>0.67</td>
<td>0.57</td>
<td>6.59</td>
</tr>
<tr>
<td>all in</td>
<td>20.39</td>
<td>-18.31</td>
<td>19.20</td>
<td>22.20</td>
<td>0.54</td>
<td>0.16</td>
<td>0.86</td>
<td>12.15</td>
</tr>
<tr>
<td>Panel C: With ( w_{it} = I(c_{it} &lt;</td>
<td>FD_{it}</td>
<td>) \times \left[ \theta T x_{it}/N_t \right] )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fd</td>
<td>12.83</td>
<td>-20.70</td>
<td>11.91</td>
<td>17.18</td>
<td>2.66</td>
<td>-0.89</td>
<td>0.69</td>
<td>8.35</td>
</tr>
<tr>
<td>mom</td>
<td>6.67</td>
<td>-7.01</td>
<td>2.14</td>
<td>6.04</td>
<td>2.37</td>
<td>-0.07</td>
<td>0.35</td>
<td>4.33</td>
</tr>
<tr>
<td>rev</td>
<td>3.44</td>
<td>-3.84</td>
<td>-0.37</td>
<td>3.00</td>
<td>4.66</td>
<td>-0.16</td>
<td>-0.12</td>
<td>2.36</td>
</tr>
<tr>
<td>sign</td>
<td>18.10</td>
<td>-23.09</td>
<td>12.08</td>
<td>20.23</td>
<td>2.74</td>
<td>-0.76</td>
<td>0.60</td>
<td>5.98</td>
</tr>
<tr>
<td>all in</td>
<td>26.70</td>
<td>-22.75</td>
<td>28.48</td>
<td>26.84</td>
<td>0.69</td>
<td>-0.16</td>
<td>1.06</td>
<td>16.69</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\alpha & \quad \beta_1 & \quad \beta_2 & \quad \beta_3 & \quad \beta_4 & \quad \text{Adj-Rsquared} \\
2.08 & \quad 1.31 & \quad - & \quad - & \quad - & \quad -0.10 \\
[3.20] & [0.90] & & & & \\
r_{\text{opt},t} = \alpha + \beta_1 \Delta \text{cons}_{t} + \varepsilon_t \\
2.38 & \quad -0.06 & \quad - & \quad - & \quad - & \quad 3.76 \\
[4.31] & [-2.90] & & & & \\
r_{\text{opt},t} = \alpha + \beta_1 \Delta TED_{t} + \varepsilon_t \\
2.38 & \quad -0.46 & \quad - & \quad - & \quad - & \quad 7.27 \\
[4.41] & [-3.98] & & & & \\
r_{\text{opt},t} = \alpha + \beta_1 \Delta VIX_{t} + \varepsilon_t \\
2.37 & \quad 26.80 & \quad - & \quad - & \quad - & \quad -0.42 \\
[4.21] & [0.45] & & & & \\
r_{\text{opt},t} = \alpha + \beta_1 \Delta c_{t} + \varepsilon_t \\
2.38 & \quad -8.71 & \quad - & \quad - & \quad - & \quad 1.88 \\
r_{\text{opt},t} = \alpha + \beta_1 \Delta \sigma_{FX,t} + \varepsilon_t \\
2.19 & \quad 0.44 & \quad - & \quad - & \quad - & \quad 7.43 \\
[4.03] & [4.02] & & & & \\
r_{\text{opt},t} = \alpha + \beta_1 \text{RMRF}_{t} + \beta_2 \text{SMB}_{t} + \beta_3 \text{HML}_{t} + \beta_4 \text{WML}_{t} + \varepsilon_t \\
2.07 & \quad 0.50 & \quad 0.01 & \quad 0.20 & \quad 0.09 & \quad 6.88 \\
[3.74] & [4.08] & [0.05] & [1.19] & [0.87] & - \\
r_{\text{opt},t} = \alpha + \beta_1 \text{RX}_{t} + \beta_2 \text{HML}_{FX,t} + \varepsilon_t \\
1.73 & \quad 0.32 & \quad 1.35 & \quad - & \quad - & \quad 20.85 \\
\end{align*}
\]

Table 4. Risk exposures of the optimal strategy. We regress the OOS returns of the optimal strategy (after transaction costs) on each set of risk factors. The optimal strategy uses the forward discount, its sign, 3-month momentum and reversal as conditional variables. Standard OLS coefficients (and t-statistics in brackets). The OOS returns are from 1996:03 to 2011:12. ‘RMRF’ is the return on the market minus the return on the risk-free rate. ‘SMB’ is the return of small stocks minus the return of large stocks. ‘HML’ is the return of high book-to-market stocks (value) minus the return of low book-to-market stocks (growth). ‘WML’ is the return of previous winners minus the return of previous losers in the stock market.
Table 5. Time-varying risk of the optimal strategy. In each row we regress the OOS returns, after transaction costs, of the optimal strategy on the market and bond returns, using a different risk proxy as a state variable to account for time-varying risk exposure. We standardize all risk proxies subtracting the mean and dividing by the standard deviation. Standard OLS coefficients and t-statistics (in brackets). The optimal strategy uses sign, fd, momentum and reversal and re-estimates the coefficients in the OOS period every month. The OOS returns are from 1996:03 to 2011:12. ‘RMRF’ is the return on the stock market minus the risk-free rate. The return on bonds is the return of the 10-year US bond minus the risk-free rate.
<table>
<thead>
<tr>
<th>Strategy</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>STD</th>
<th>Kurt.</th>
<th>Skew</th>
<th>SR</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>fd, mom, rev and sign</td>
<td>26.70</td>
<td>-22.75</td>
<td>28.48</td>
<td>26.84</td>
<td>0.69</td>
<td>-0.16</td>
<td>1.06</td>
<td>16.69</td>
</tr>
<tr>
<td>Stock market</td>
<td>7.16</td>
<td>-14.94</td>
<td>3.17</td>
<td>12.46</td>
<td>1.36</td>
<td>-0.81</td>
<td>0.25</td>
<td>2.83</td>
</tr>
<tr>
<td>Stock market+curr.</td>
<td>27.27</td>
<td>-21.87</td>
<td>27.95</td>
<td>26.93</td>
<td>0.73</td>
<td>-0.16</td>
<td>1.04</td>
<td>16.07</td>
</tr>
<tr>
<td>FF factors</td>
<td>19.89</td>
<td>-29.96</td>
<td>12.94</td>
<td>27.41</td>
<td>1.53</td>
<td>-0.84</td>
<td>0.47</td>
<td>-1.53</td>
</tr>
<tr>
<td>FF factors+curr.</td>
<td>31.75</td>
<td>-22.26</td>
<td>27.06</td>
<td>25.79</td>
<td>1.32</td>
<td>0.13</td>
<td>1.05</td>
<td>16.83</td>
</tr>
<tr>
<td>Carhart factors</td>
<td>33.51</td>
<td>-63.23</td>
<td>20.84</td>
<td>35.67</td>
<td>8.02</td>
<td>-1.37</td>
<td>0.58</td>
<td>-30.46</td>
</tr>
<tr>
<td>Carhart factors+curr.</td>
<td>19.29</td>
<td>-24.21</td>
<td>15.78</td>
<td>22.92</td>
<td>0.94</td>
<td>-0.45</td>
<td>0.69</td>
<td>7.58</td>
</tr>
<tr>
<td>Stocks and bonds</td>
<td>8.13</td>
<td>-13.74</td>
<td>5.39</td>
<td>12.16</td>
<td>2.15</td>
<td>-0.93</td>
<td>0.44</td>
<td>5.19</td>
</tr>
<tr>
<td>Stock and bonds+curr.</td>
<td>23.53</td>
<td>-22.67</td>
<td>27.98</td>
<td>27.47</td>
<td>0.80</td>
<td>-0.28</td>
<td>1.02</td>
<td>15.19</td>
</tr>
</tbody>
</table>

Table 6. The OOS performance of portfolios combining a currency strategy with different background assets. The currency strategy uses momentum, the interest rate spread, reversal and sign. Each row denoted with ‘+curr.’ combines the available factors with the currency strategy. Results with transaction costs. Optimizations carried out with a CRRA of 4 and 240 months in the initial in-sample estimate. ‘FF’ stands for the Fama-French factors. The bonds return is the excess return of the 10 year US bond over the risk-free rate. The Carhart factors are the same as the Fama-French factors but also include stock momentum (‘WML’). The OOS period is from 1996:03 to 2011:12. The max and the min are, respectively, the maximum and the minimum one-month return in the sample, expressed in percentage points. The mean is the annualized average return, in percentage points. The standard deviation and Sharpe ratio are also annualized and “Kurt.” stands for excess kurtosis. The certainty equivalent is expressed in annual percentage points. It is the constant return that would provide the same utility as the series of returns of the given strategy.
Figure 1. The performance of Deutsche Bank currency ETFs (in euros). Each line plots the cumulative monthly returns of a Deutsche Bank ETF (in euros) from 2008:01 to 2011:12.
Figure 2. The estimates of the coefficients of the portfolio in the OOS period from 1996:03 to 2011:12. Optimization with CRRA of 4 and considering transaction costs.
Figure 3. The OOS value of currency strategies for investors exposed to different background risks. Each set of columns shows the performance of an optimized portfolio with the available assets (light grey) and one which combines it with the currency strategy (dark grey). The currency strategy uses the information on the interest rate spread, sign, momentum and reversal. The background assets are the return on the stock market minus the risk free rate (‘RMRF’), the Fama-French stock market factors market, size and value (‘FF’), these augmented with stock momentum (‘Carhart’), and the return of the stock market and 10-year bonds (‘S+B’). The OOS period is from 1996:03 to 2011:12. Results with transaction costs.